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SOIL-STRUCTURE INTERACTION ANALYSIS USING FIXED-BASE STRUCTURAL MODES IN SASSI

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ABSTRACT

Frequency-domain analysis of soil/structure systems using fixed-base structural modes offers a great advantage in significantly reducing the computer runtime for probabilistic soil-structure interaction (SSI) analysis following the Latin Hypercube Simulation (LHS) procedure documented in NUREG/CR-2015. In the LHS approach, a series of SSI simulations are performed to statistically account for the uncertainty in the ground motion specifications as well as the structure and soil properties (stiffness and damping) in the response. In general, 30 to 60 simulations are performed for a combination of parameters for each simulation selected based on the LHS design.

This paper presents the methodology for SSI analysis in the frequency domain using fixed-base structural modes implemented in MTR/SASSI (Tabatabaie, 2014). The methodology is robust and very effective for analyzing structures supported on foundations that can be assumed to be rigid, which is often the case for practical applications related to nuclear power plant (NPP) designs. First, the rigid foundation impedance and scattering properties are calculated, followed by the modal properties of the fixed-base structure which can be calculated directly in MTR/SASSI or imported from elsewhere. The results of the above analyses are then synthesized to calculate the response of the total soil/structure system, as described in this paper. This process is repeated for each simulation with specified input motions and randomized soil and structural properties. Using the above "hybrid modal" method, the response of a soil/structure system can be obtained at significantly reduced computer runtime that it would take to perform analysis of the full SSI model for each simulation. The accuracy and effectiveness of this hybrid modal SSI method is demonstrated using two example problems of a surface and an embedded NPP structure.

METHODOLOGY

The dynamic response of the structure, including the SSI effects, can be written as follows:

$$U = Y + T \, . \, U_b \tag{1}$$

Where U is nx1 vector of absolute displacements of the structure, Y is nx1 vector of structure displacements relative to the foundation, U_b is 6x1 displacements of rigid foundation center, and T is nx6 rigid body transformation matrix.

The equilibrium equation of the foundation mass can be written as follows:

$$M_b \, . \, U_b'' + X_g \, (U_b - U_g) = P_b \tag{2}$$

Where U_g is 6x1 vector of kinematic displacements at center of rigid foundation obtained from scattering analysis, P_b is 6x1 vector of foundation driving force, M_b and X_b are 6x6 matrices of rigid foundation mass and impedance referred to the foundation center.

With $P_b = -T^T. \, M \, . \, U''$ where M is nxn mass matrix of structure, Equations 1 and 2 can be combined and rearranged which will result in Equation 3 as shown below:

$P_b = -T^T . M (Y'' + T . U_b'') = -T^T . M . Y'' – T^T . M. T . U_b''$

$M_b . U_b'' + X_g (U_b – U_g) = -T^T . M . Y'' – T^T . M . T . U_b''$

$(M_b + T^T . M . T) U_b'' + T^T . M . Y'' + X_g . U_b = X_g . U_g$ (3)

For the structure, the equilibrium equation can be written as follows:

$M . Y'' + C . Y' + K . Y = -M . T. U_b''$ (4)

Where K and M are nxn matrices of structure stiffness and damping. Assuming normal modes of the structure on a fixed base condition exist, we can write:

$Y = Ø . A$ (5)

Where Ø is nxm matrix of mode shapes and A is mx1 vector of modal coordinate of structure and m is the number of modes. With this, Equation 4 can be written as follows:

$M . Ø . A'' + C . Ø . A' + K . Ø . A = -M . T. U_b''$

$Ø^T. M . Ø . A'' + Ø^T. C . Ø . A' + Ø^T. K . Ø . A = - Ø^T. M . T. U_b''$

$m_j . A_j'' + c_j . A_j' + k_j . A_j = - Ø_j^T. M . T. U_b''$ (6)

Where m_j, c_j and k_j are mxm matrices of modal mass, damping and stiffness of the structure, and $Ø_j$ and A_j are 1xm vector of mode shape and modal coordinate for the j-th mode. Letting $Ω_j$ and $ξ_j$ be modal circular frequency and damping ratio of the j-th mode, we can write:

$Ω_j = \sqrt{(k_j / m_j)}$

$k_j = m_j . Ω_j^2$

$ξ_j = c_j / c_{cr} = c_j /(2 \sqrt{(k_j . m_j)} = c_j / (2 m_j . Ω_j)$

$c_j = 2 ξ_j . m_j . Ω_j$

Substituting above parameters in Equation 6, we get:

$m_j . A_j'' + 2 ξ_j . m_j . Ω_j . A_j' + m_j . Ω_j^2 . A_j = - Ø_j^T. M . T. U_b''$

$A_j'' + 2 ξ_j . Ω_j . A_j' + Ω_j^2 . A_j = (- Ø_j^T. M . T / m_j) . U_b''$ (7)

Assume $A_j = a_j e^{i ω t}$ where ω is circular frequency and $i = \sqrt{-1}$, we can solve for a_j from Equation 7.

$(-ω^2 + 2 ξ_j . Ω_j . ω . i + Ω_j^2) a_j = (- Ø_j^T. M . T / m_j) . (-ω^2. U_b)$

$a_j = [ω^2 / (Ω_j^2 - ω^2 + 2 ξ_j . Ω_j . ω . i)] . (Ø_j^T. M . T / m_j) . U_b$ (8)

Let:

$H_j = ω^2 / (Ω_j^2 - ω^2 + 2 ξ_j . Ω_j . ω . i)$

$Q_j = T^T. M . Ø_j$

Then:

$$a_j = H_j \,.\, (Q_j^T / m_j) \,.\, U_b \quad (9)$$

With $y_j = Ø_j \,.\, a_j$ and replacing a_j from Equation 9, we get:

$$y_j = Ø_j \,.\, H_j \,.\, (Q_j^T / m_j) \,.\, U_b \quad (10)$$

Because H_j is a single value, Equation 10 can be rearranged as follows:

$$y_j = H_j \,.\, (Ø_j \,.\, Q_j^T / m_j) \,.\, U_b \quad (11)$$

Let's rewrite Equation 3 the frequency domain:

$$(M_b + T^T.\, M \,.\, T)\, (-\omega^2.\, U_b) + T^T.\, M \,.\, (-\omega^2.\, Y) + X_g \,.\, U_b = X_g \,.\, U_g$$
$$[X_g - \omega^2.\, (M_b + T^T.\, M \,.\, T)] \,.\, U_b - \omega^2.\, T^T.\, M \,.\, Y = X_g \,.\, U_g \quad (12)$$

with $Y = \sum y_j$ and replacing y_j from Equation 11, Equation 12 is written as follows:

$$[X_g - \omega^2.\, (M_b + T^T.\, M \,.\, T)] \,.\, U_b - \omega^2.\, T^T.\, M \,.\, \sum[H_j \,.\, Ø_j \,.\, Q_j^T / m_j] \,.\, U_b = X_g \,.\, U_g$$
$$[X_g - \omega^2.\, (M_b + T^T.\, M \,.\, T)] \,.\, U_b - \omega^2.\, \sum(H_j \,.\, Q_j \,.\, Q_j^T / m_j) \,.\, U_b = X_g \,.\, U_g$$
$$\{X_g - \omega^2.\, [M_b + T^T.\, M \,.\, T + \sum(H_j \,.\, Q_j \,.\, Q_j^T / m_j)]\} \,.\, U_b = X_g \,.\, U_g \quad (13)$$

Let $\boldsymbol{M}$ be equivalent dynamic mass inertia of the structure defined as follows:

$$\boldsymbol{M} = M_b + T^T.\, M \,.\, T + \sum(H_j \,.\, Q_j \,.\, Q_j^T / m_j) \quad (14)$$

Substituting $\boldsymbol{M}$ from Equation 14 into Equation 13 results in Equation 15 below:

$$(X_g - \omega^2.\, \boldsymbol{M}) \,.\, U_b = X_g \,.\, U_g \quad (15)$$

Solution of Equation 15 yields the base motion, U_b. With the base motion known, the relative response of the system, Y, can be calculated from Equation 16 below:

$$Y = \sum[H_j \,.\, Ø_j \,.\, Q_j^T / m_j] \,.\, U_b \quad (16)$$

And the total response, U, can then be calculated from Equation 1.

SUMMARY OF ANALYSIS PROCEDURE

1. Perform modal analysis of the structure on fixed foundation and output modal properties of the structure, i.e. modal frequencies Ω_j, mode shapes $Ø_j$, and participation factors for significant number of modes m.
2. Calculate rigid massless foundation impedance X_g and scattering U_g properties referenced at the foundation center.
3. Calculate foundation total response U_b at the foundation center from:

$$(X_g - \omega^2.\, \boldsymbol{M}) \,.\, U_b = X_g \,.\, U_g$$

Where:

$\boldsymbol{M} = M_b + T^T . M . T + \sum(H_j . Q_j . Q_j^T / m_j)$

$H_j = \omega^2 / (\Omega_j^2 - \omega^2 + 2\, \xi_j . \Omega_j . \omega . i)$

$Q_j = T^T . M . Ø_j$

$m_j = Ø_j^T . M . Ø_j$

$\sum$ = Summation over no. of modes m

4. Solve for structural relative response Y from:

   $Y = \sum[H_j . Ø_j . Q_j^T / m_j] . U_b$

5. Solve for the total structural response U from:

   $U = Y + T . U_b$

VERIFICATION OF METHODOLOGY

To demonstrate the accuracy of the above procedure, two example problems – one for an embedded NPP structure and one for a ground surface-supported NPP structure – are presented below.

Example 1: Embedded NPP Structure

Seismic response of a ¼-scale reactor containment experimental model in Lotung, Taiwan (Tang, et al., 1990) was calculated using the hybrid modal as well as direct SSI analysis methods in MTR/SASSI (Tabatabaie, 2013). The results of the two analyses, together with those of the actual recorded motions at several key locations in the structure, were compared to validate the accuracy of the hybrid modal method. Figure 1 shows the geometry of the structure. The soil profile and properties are shown in Figure 2. The soil properties were strain-dependent; therefore, no further adjustment of the soil properties was performed to account for the site response and SSI effects.

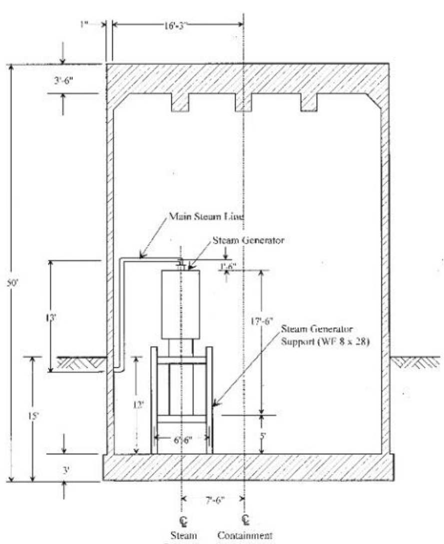


Figure 1. Cut-Away Section of Structure

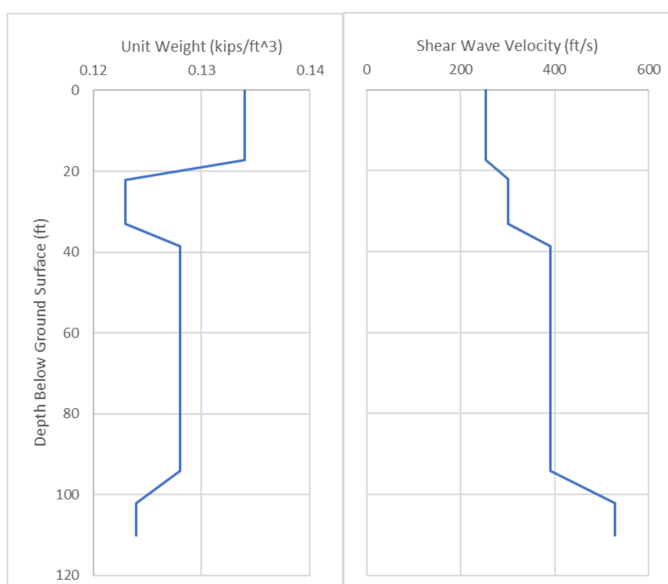


Figure 2. Soil Profile and Properties

The seismic environment was characterized primarily consisting of vertically propagating shear and compressional waves with the input motion specified at the free-field ground surface. The acceleration time history of the input motion and its 5%-damped response spectra are shown in Figure 3. The input motion corresponds to the horizontal (EW) component of the earthquake motions recorded at the site.

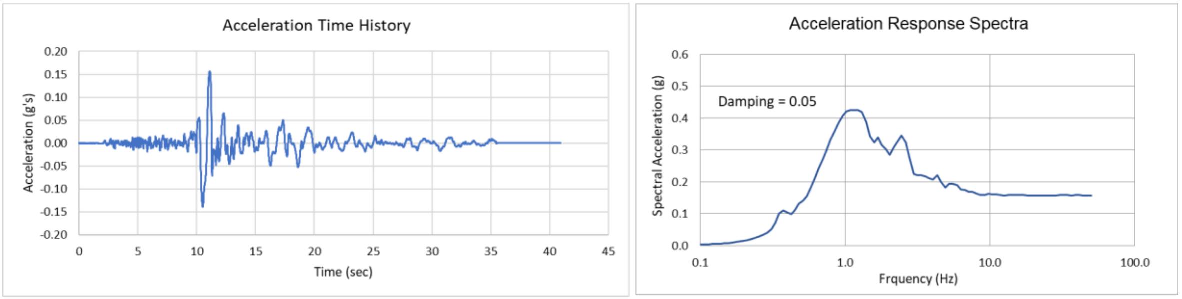


Figure 3. Acceleration Time History and Response Spectra of Input Motion

The finite element (FE) model of the structure is shown in Figure 4a. Because of symmetry, only one-half cutaway view of the structure is shown to better show the internal details of the structure. The containment shell, internal structures, and piping were modeled by beam elements with lumped masses; the basement walls were modeled by shell elements; and the base slab was modeled by solid elements. The shell elements were extended to the bottom of the base slab to permit proper moment transfer between the base slab and basement wall. The mass of the basement wall and base slab were lumped to the containment beam elements. The excavated soil mass was modeled by solid elements, as shown in Figure 4b.

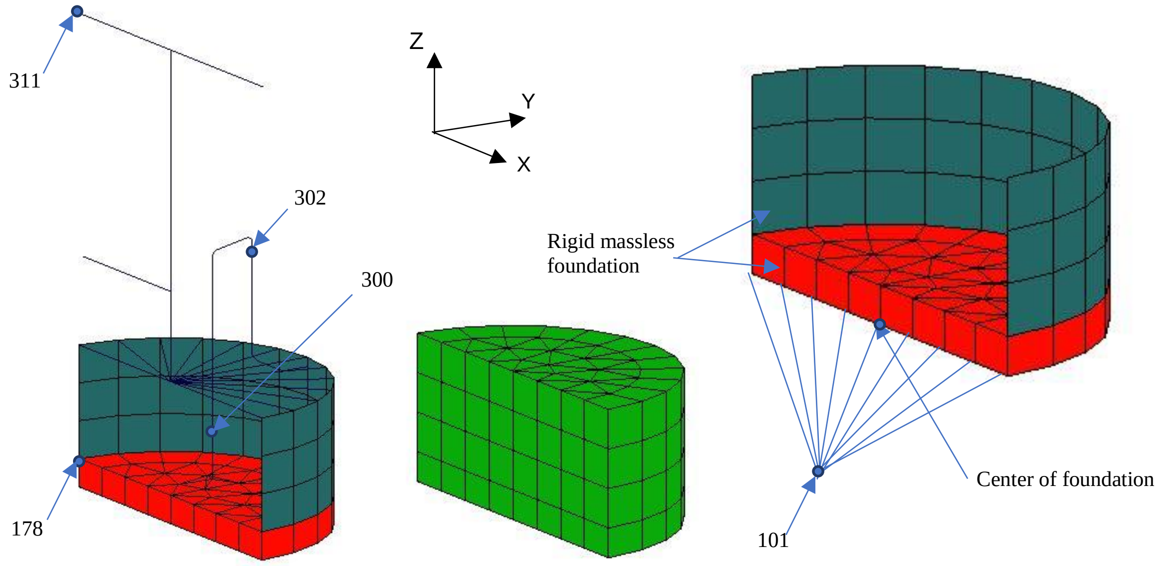
For the direct method analysis, the soil/structure system in Figure 4a was subjected to the free-field input motion in the horizontal X-direction, and the response of the total SSI system was calculated using the direct method in MTR/SASSI. The hybrid modal analysis was performed in MTR/SASSI, as described in the following four steps:

Step 1: The foundation scattering properties were calculated using the rigid massless foundation model shown in Figure 4c and the excavated soil model shown in Figure 4b. For this analysis, all nodes at the bottom of the foundation base slab were constrained to a Master Node 101 with 6 degrees of freedom (DOF) at the bottom center of foundation (see Figure 4c). Master Node 101 provides a convenient reference point for coupling the results of the foundation and structural analysis discussed later. The above model was then subjected to the free-field motion as discussed earlier, and 6x1 foundation scattering (kinematic) properties in terms of the horizontal, vertical and rocking responses versus frequency at Node 101 were calculated.

Step 2: The foundation impedance properties were calculated using the same rigid massless foundation and soil model used in the scattering analysis – except that instead of applying the free-field motion, the model was subjected to unit amplitude forces and moments at Node 101. The results of the analysis in terms of calculated displacements and rotations at Node 101 were then used to calculate the 6x6 foundation impedance matrices in terms of foundation stiffness and damping versus frequency.

Step 3: The modal properties of the structure on a fixed-base condition were calculated in terms of its modal frequencies, mode shapes and mode participation factors. For this model, all the modes of the structure were extracted. The structural model used in the hybrid modal analysis is the same as the one shown in Figure 4a except that all the nodes at the bottom of the foundation were constrained to Master Node 101 at the foundation center, as was the case for the scattering and impedance models but with all 6 DOFs fixed.

Step 4: The modal properties of the fixed-base structure were synthesized with the results of the foundation scattering and impedance properties to calculate the response of the total SSI system, as presented above.



a) Structure b) Excavated Soil c) Rigid Foundation

Figure 4. Finite Element Models used in MTR/SASSI SSI Analysis

The response of the structure in terms of its 5%-damped acceleration response spectra at the top of the foundation base slab (Node 178), the base of the steam generator (Node 300), the top of the steam generator (Node 302), and the top of the containment structure (Node 311) calculated from the direct SSI and hybrid modal analyses, together with those of the actual recorded motions, are compared in Figure 5. As shown in Figure 5, there is excellent agreement between the calculated results from the direct and hybrid modal SSI analyses. The results also compare reasonably well with the actual recorded data.

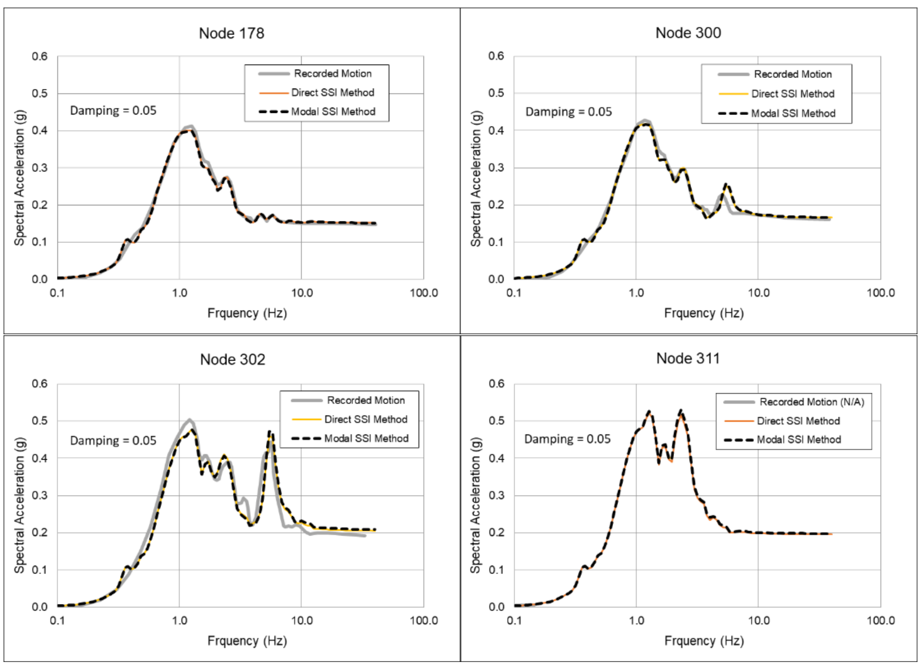


Figure 5. Comparison of 5%-Damped Acceleration Response Spectra Between
Direct and Hybrid Modal SSI Methods and Recorded Motions

Example 2: Surface-Supported NPP Structure

In this example, the seismic SSI response of a large nuclear building supported at shallow depth below ground surface was calculated using both the hybrid modal and direct methods in MTR/SASSI. The results were then compared to evaluate the accuracy of the hybrid modal method, as described above. The results of the direct analysis method were used as the benchmark solution.

The building is a concrete box type structure consisting of concrete shear walls, floor slabs and roofs. The structure has a medium size footprint with a shallow embedment of 5 feet. The base slab incorporates 6-foot-deep shear keys to improve the foundation stability against sliding. Figure 6 shows the overall configuration and dimensions of the building.

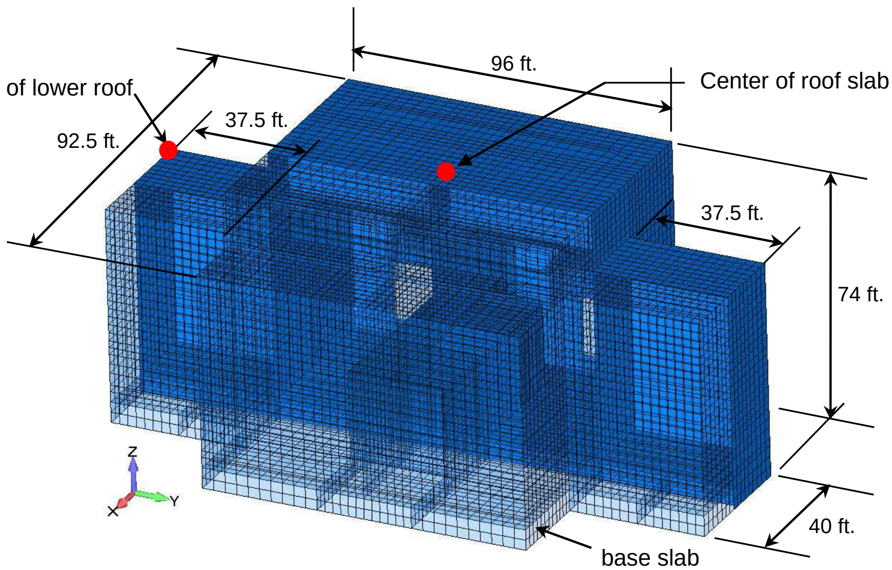


Figure 6. Finite Element Model of Structure

The foundation is supported on a semi-infinite, uniform medium stiff soil profile. The dynamic soil properties consist of density, γ = 110 lb/ft^3, shear wave velocity, V_s = 1,600 ft/sec, Poisson's ratio, ν = 0.377, and material damping, β = 0.04. The soil properties were strain compatible.

The seismic environment consisted of vertically propagating shear waves with the input motion specified at the free-field ground surface. The acceleration time history of the input motion and its 5%-damped acceleration response spectra are shown in Figure 7.

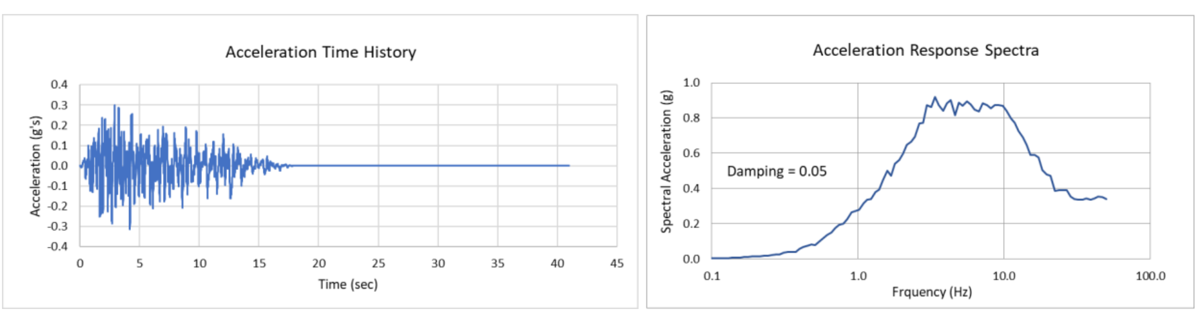


Figure 7. Acceleration Time History and Response Spectra of Input Motion

A detailed FE model of the building was used in this study, as shown in Figure 6. The FE model incorporates all the major details of the structure. The mass of the shear keys is lumped with the mass of the base slab nodes. The FE model consists of plate/shell elements representing the concrete walls, roof, and base slab – all of which are modelled as flexible members with appropriate section properties.

The total soil/structure system was first analyzed using the direct SSI method in MTR/SASSI. The relatively shallow embedment was ignored; therefore, there is no excavated soil model. The structure was subjected to input motion in the X-direction and analysis was performed to frequencies up to 40 Hz.

For the hybrid modal analysis, two separate foundation impedance and structural modal analyses were performed, and the results were synthesized, as described below. It is noted that since the structure is surface-supported, foundation scattering analysis is not required because the kinematic motion of the rigid massless foundation is equal to the input motion.

Foundation Impedance Analysis: For this analysis, a separate soil/structure FE model of the rigid massless foundation slab was developed, as shown in Figure 8. The 6x6 foundation impedance matrix was then calculated in reference to the foundation center using the RIMP module of MTR/SASSI. The RIMP procedure for calculating rigid foundation impedance is rigorous, fast and efficient since it takes advantage of rigid-body motion constraints, thus bypassing inversion of a large, complex-valued, fully coupled subgrade flexibility matrix in SASSI (Lysmer, et al., 1981). The results of the foundation impedance properties in terms of normalized foundation stiffness and damping versus wave number $a_o = \omega.R/V_s$ for horizontal X, vertical Z and rocking YY components are shown in Figure 9, where ω is circular frequency, R is equivalent foundation radius, and V_s is shear wave velocity of foundation soil.

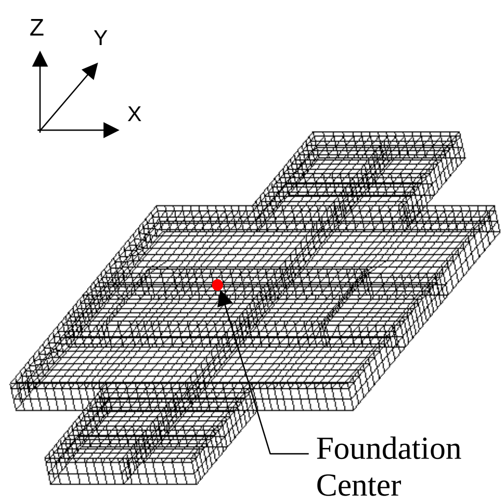


Figure 8. Rigid Foundation Model

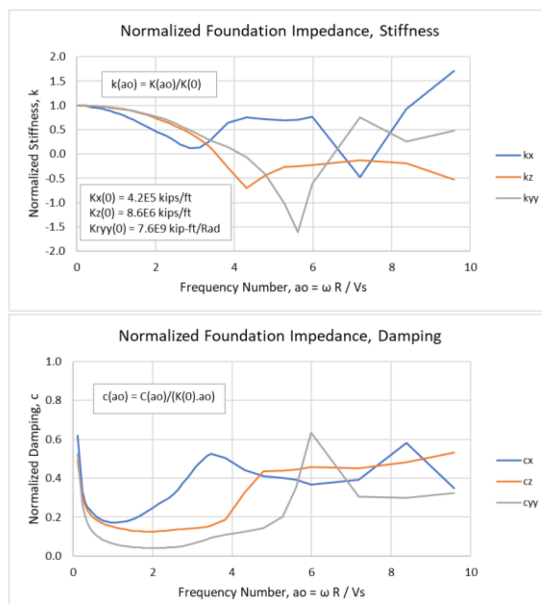


Figure 9. Foundation Impedance Properties

Hybrid Modal Analysis: This analysis consisted of first calculating the modal properties of the structure on a fixed-base condition and then synthesizing the results with those of the foundation impedance properties developed above to calculate the response of the total SSI system. The structural model used in the hybrid modal analysis is the same as the one shown in Figure 6, except that to impose foundation base fixity and provide proper coupling with the foundation impedance, all the nodes at the bottom of the foundation were constrained to a fixed master node defined at the reference foundation center (see Figure 8). A total of 1,120 modes of the fixed-base structure were extracted in MTR/SASSI with a total cumulative mass participation factor of about 97.5 percent in all three X-, Y- and Z-directions. Several significant modes of the structure, together with their effective mass participation factors, are tabulated in Table 1.

Table 1. Significant Modal Frequencies and Effective Mass Participation Factors

Freq (Hz)	MPF_X	MPF_Y	MPF_Z	Freq (Hz)	MPF_X	MPF_Y	MPF_Z
10.2	0.0	73.6	0.0	17.0	0.1	0.0	35.2
10.8	69.2	0.0	0.0	19.0	1.9	0.0	2.7
12.3	1.6	0.0	0.4	22.0	0.0	4.5	0.0
13.3	3.2	0.0	0.2	28.0	0.0	2.0	0.0
14.1	0.0	0.0	3.9	31.8	0.3	0.0	2.7
16.0	0.0	3.8	0.0	32.9	0.3	0.0	2.4

The results of the direct and hybrid modal SSI analyses in terms of 5%-damped acceleration response spectra calculated in the X- and Z-directions at the center of the roof slab and corner of the lower roof are compared in Figure 10. As shown in Figure 10, there is excellent agreement between the calculated results from the two methods. This validates the accuracy of the hybrid modal method for analysis of large SSI systems. In terms of the computer runtime, the hybrid modal analysis took a fraction of time that required for the direct SSI analysis.

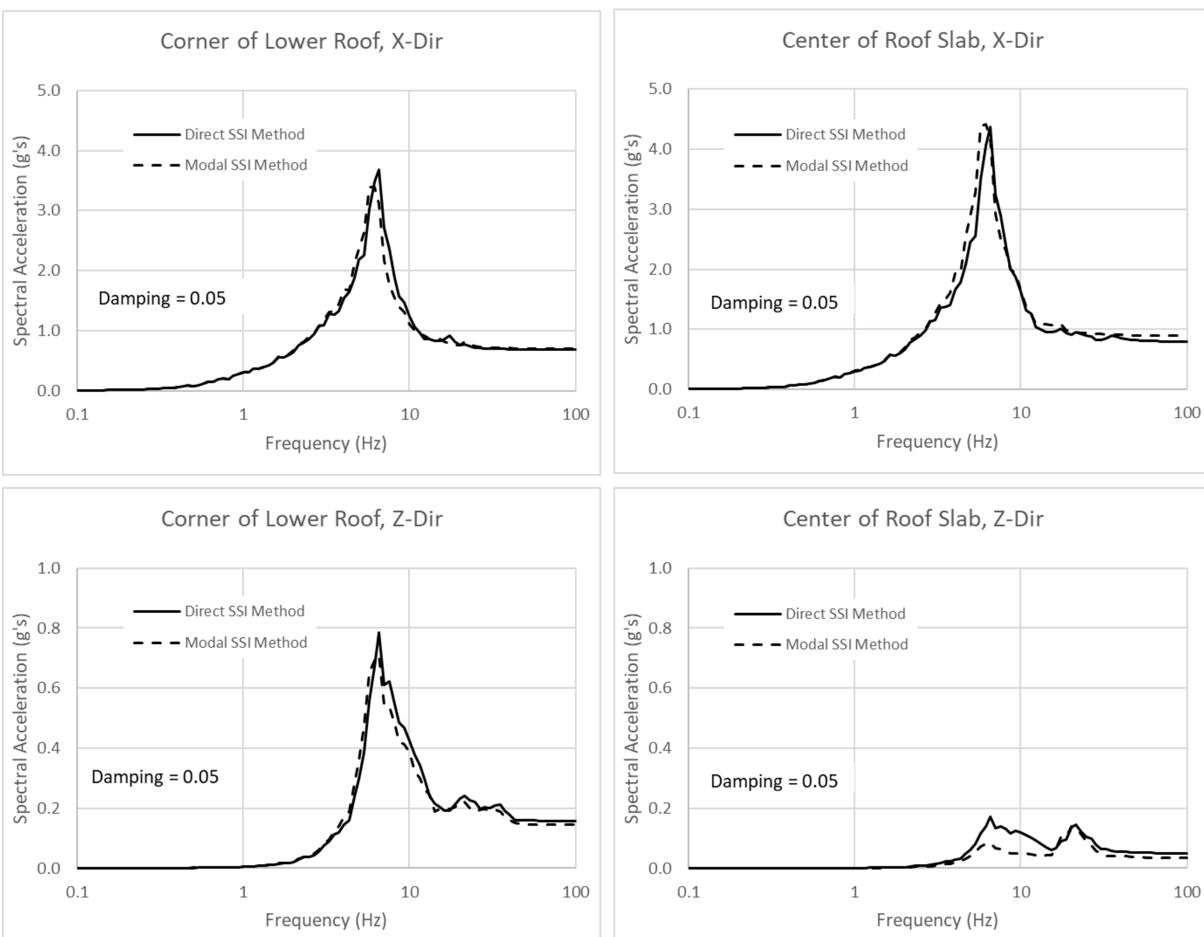


Figure 10. Comparison of 5%-Damped Acceleration Response Spectra

CONCLUSIONS

- Hybrid modal SSI analysis offers a fast and efficient method for seismic response analysis of large-scale and complex structures founded on rigid foundations, which is typical of NPP designs.
- The methodology is valid for linear systems for which the principle of superposition applies. This is a reasonable assumption for SSI analysis of NPP structures where the structure is modelled as linear elastic, and the soil properties are strain-compatible characterized by the equivalent linear method in a seismic response analysis.
- With the above considerations, the hybrid modal SSI analysis can be an ideal alternative method implemented in MTR/SASSI for repetitive analysis, generally in the range of 30 to 60 large-scale soil/structure model simulations, such as in the Latin Hypercube method.
- Formulations for implementing the hybrid modal analysis method in SASSI were presented. The accuracy of this method was validated by calculating the seismic SSI response of a surface-supported and an embedded NPP structure with rigid foundations using both the direct and hybrid modal methods in MTR/SASSI and comparing the results. For the latter example problem, the results were also compared against actual recorded data.
- Excellent agreement between the results of the direct and hybrid modal methods as well as with the actual recorded motions in the structure from the two example problems validates the accuracy of the hybrid modal method for SSI application in MTR/SASSI.
- In terms of the computer runtime, the hybrid modal analysis takes significantly less time compared to the direct SSI analysis. This makes hybrid modal method very attractive when large number of soil/structure simulations are required in SASSI SSI analysis.

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