TECHNICAL NOTE

The Accuracy of Subtraction Method Examined in MTR/SASSI (Part 2)
June 20, 2011
MTR & Associates

The scattering and compliance functions for a rigid, massless, cylindrical foundation fully embedded in a uniform halfspace are used to examine the accuracy of the Flexible Volume Sub-structuring (FVS) method for seismic SSI analysis using the Direct, Subtraction and Modified Subtraction Models. Available solutions to this problem are reported in Ref. [1, 2]. This is considered a "benchmark problem".

Problem Description

Figure 1 shows the foundation model and properties. The foundation has an embedment ratio of R/H = 1, where R and H are the radius and depth of foundation, respectively. The properties for this problem are dimensionless. The halfspace has a damping of 1% for shear waves and 0.5% for compression waves.

The computer program MTR/**SASSI** [3] is used to calculate the foundation scattering and impedance functions via the Direct, Subtraction and Modified Subtraction Models. The scattering properties of the foundation are obtained for vertically propagating plane shear waves with control motion assigned at the free-field surface, and the results are compared to those reported in Ref. [1]. The foundation compliance functions include the horizontal, vertical, rocking, and coupled horizontal-rocking responses. The calculated foundation compliance functions are compared with those reported in Ref. [2]. The scattering and impedance functions refer to the bottom center of the foundation.

MTR/SASSI Model

The finite element model analyzed by MTR/**SASSI** is shown in Figure 2. Due to symmetry about the xz- and yz-planes, only one-quarter of the foundation is modeled. The foundation and excavated soil models consist of 8-node solid (brick) elements with uniform dimensions of 0.16667×0.16667×0.16667. The halfspace is modeled by 6 top layers and 10 added halfspace layers whose thicknesses vary with frequency. Other details of the model are shown in Figure 3 and Figure 4.

The problem is analyzed for three interaction node sets corresponding to the Direct, Subtraction and Modified Subtraction Models, as described below.

<u>Direct Model</u>: In this model, the interaction nodes include all of the excavated soil nodes. This model has 284 interaction nodes.

<u>Subtraction Model:</u> In this model, the interaction node set corresponds to the nodes located on the sides and bottom surfaces of the excavated soil model. This model has 119 interaction nodes.

<u>Modified Subtraction Model</u>: In this model, the interaction node set corresponds to the nodes on the sides, bottom and top surfaces of the excavated soil model. This model has 147 interaction nodes.

The passing frequency of the model ($f_{pass} = V_s/5h$, where h is the largest soil element size and V_s is the minimum shear wave velocity of the soil medium) is 1/5/0.16667 = 1.2 Hz. Analysis is performed to a frequency cut-off of f_{max} = 1.273 Hz, which is slightly higher than the passing frequency of the model. SV- and P-waves are assigned 1% and 0.5% damping, respectively.

Because the problem is dimensionless, the results are expressed in terms of a dimensionless frequency parameter, a_o, which is described as the ratio of foundation dimension to wave length of wave propagation.

$$a_o = 2 \pi R / \lambda$$

Where R is the foundation radius, λ is wave length and ω is circular frequency. By plugging $\lambda = V_s / f$ and $\omega = 2\pi f$ into the above equation, the maximum value of a_o corresponding to the cut-off frequency is about 8.

$$a_{o,max} = \omega R / V_s = (2\pi)(1.273)(1)/(1) = 8.$$

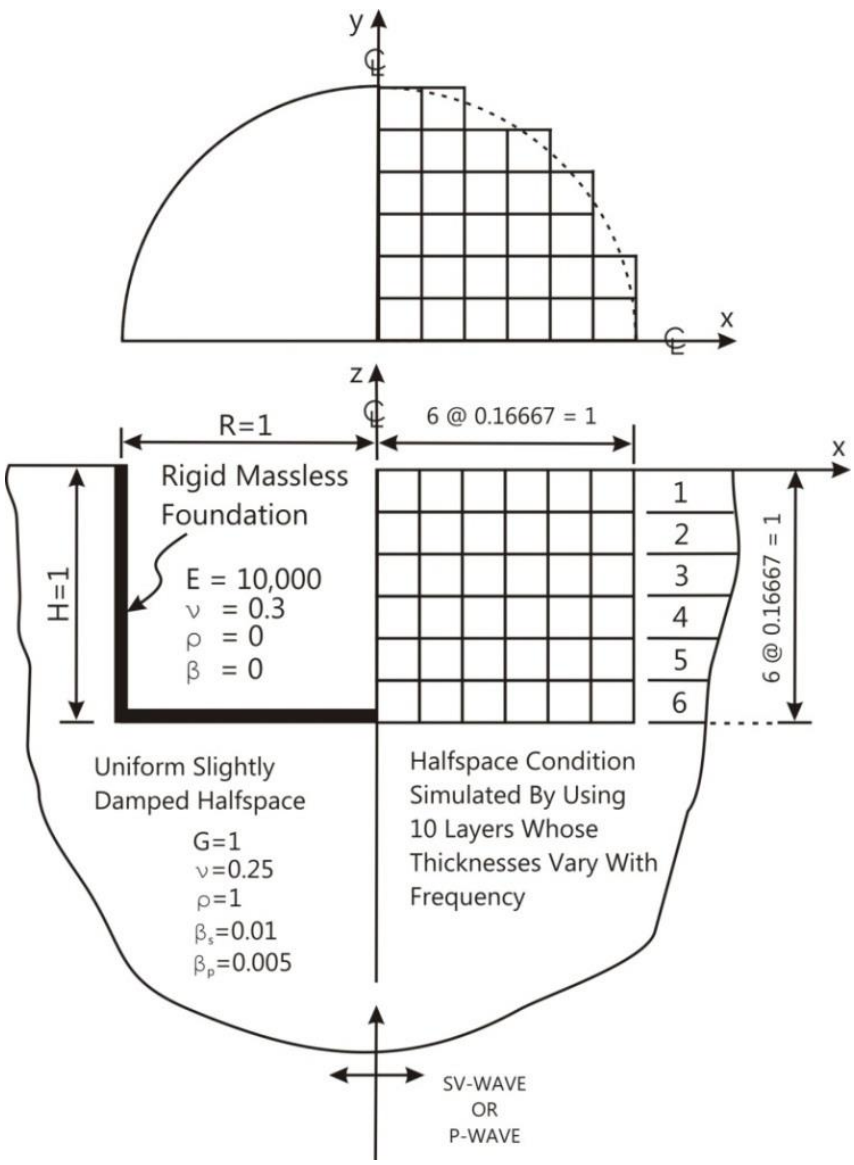


Figure 1: Foundation Model and Properties

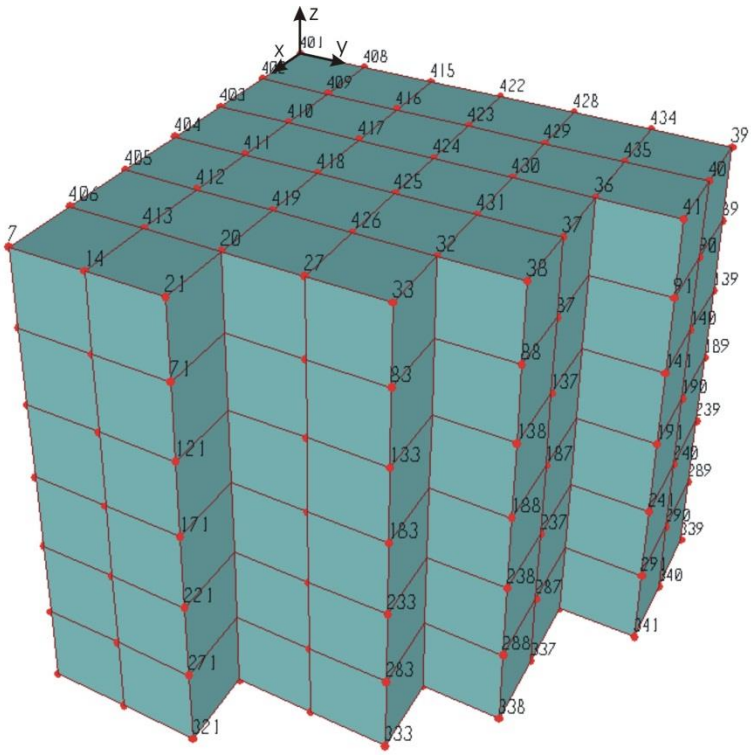


Figure 2: MTR/SASSI Finite Element Model - Foundation

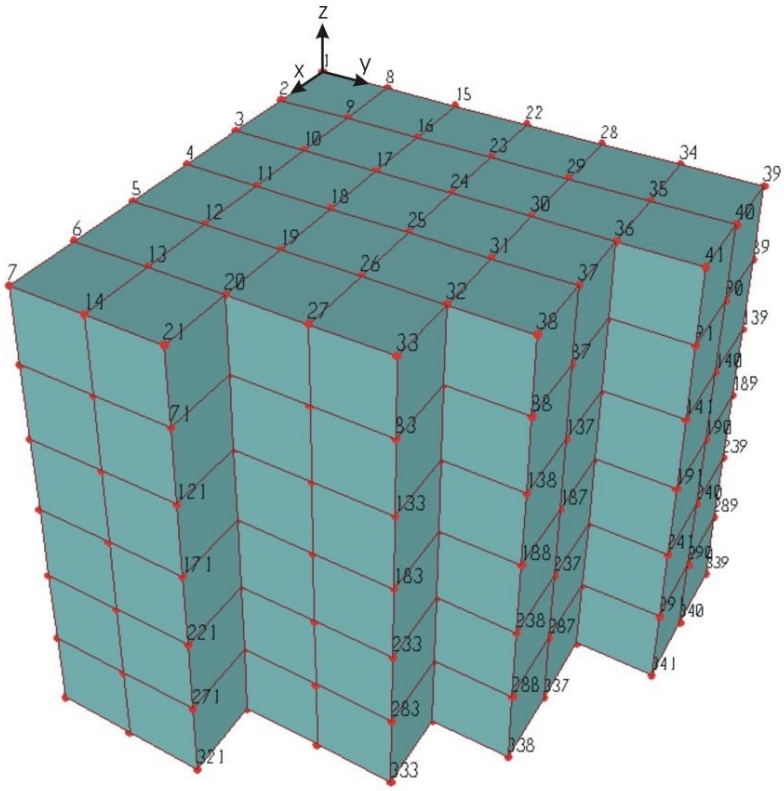


Figure 3: MTR/SASSI Finite Element Model - Excavated Soil

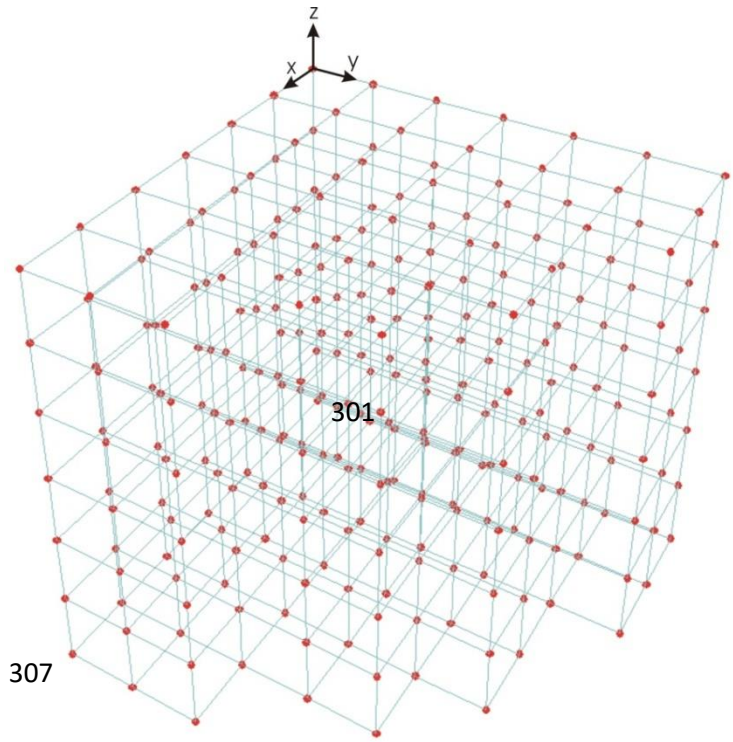


Figure 4: MTR/SASSI Finite Element Grid - Excavated Soil/Foundation

Four cases were analyzed for the Direct, Subtraction and Modified Subtraction Models, as described below.

Analysis Case 1: Foundation Scattering

Case 1 consists of calculating the horizontal (x) and rocking (yy) responses of the foundation, normalized to the input motion at the free-field surface for vertically propagating harmonic shear waves. One-quarter of the model is used by specifying xz- and yz-planes as planes of symmetry and anti-symmetry, respectively.

Analysis Case 2: Foundation Horizontal Compliance

Case 2 is similar to Case 1 with the exception that a unit amplitude harmonic force is applied to the bottom center of the foundation in the x-direction instead of subjecting the foundation to shear waves. The horizontal and vertical responses of the foundation are used to calculate the horizontal and coupled horizontal-rocking compliance functions, respectively.

Analysis Case 3: Foundation Rocking Compliance

Case 3 is identical to Case 2 with the exception that the input dynamic force is equivalent to a unit amplitude moment about the y-axis applied to the bottom center of the foundation. The corresponding

rotational and horizontal responses of the foundation are used to calculate the rocking and coupled horizontal-rocking compliance functions, respectively.

Analysis Case 4: Foundation Vertical Compliance

Case 4 consists of calculating the vertical response of the foundation subjected to a unit amplitude harmonic vertical force applied to the bottom center of the foundation. The foundation response in terms of vertical displacement calculated at the same node is used to calculate the vertical compliance function. The model is the same as that of Case 1, except that the xz- and yz-planes are both specified as planes of symmetry.

Analysis Results

The analysis results for Cases 1, 2 and 3 are derived from the horizontal and vertical displacement responses at Nodes 301 and 307 at the bottom center and bottom edge of the foundation, respectively (see **Error! Reference source not found.** 3). The results for Case 4 are derived from the vertical displacement responses at Node 301.

The results of scattering solutions (Case 1) for the Direct, Subtraction and Modified Subtraction Models are shown in Figure 5, where they are compared against those of published solutions. The results of compliance solutions (Cases 2, 3 and 4) for the same models are compared against those of the published results in Figure 6 for horizontal compliance, in Figure 7 for coupled horizontal-rocking, in Figure 8 for rocking, and in Figure 9 for vertical compliance. Note that the compliance functions are normalized and rendered dimensionless by multiplying them by $G \cdot R = 1$.

Comparison of the scattering solutions for vertically propagating SV-waves obtained from MTR/**SASSI** using the Direct and Modified Subtraction Models shows excellent agreement with those reported in the literature for all frequencies up to the maximum a_o value of 8 (see Figure 5). The results for the Subtraction Model, however, start to deviate from the published results at about $a_o = 3$.

Comparison of compliance functions obtained from MTR/**SASSI** using the Direct and Modified Subtraction Models also shows excellent agreement with the published results for all four components (i.e. horizontal, coupled horizontal-rocking, rocking and vertical -- see Figure 6 through Figure 9, respectively). Again the results for the Subtraction Model start to deviate from the published results at $a_o = 3$.

Discussion of Results

As seen in Figure 5 through Figure 9, the response transfer functions calculated using the Subtraction Model show a number of peaks and valleys at $a_o > 3$, causing significant departures from the target solution. These peaks and valleys are generally indicative of the wave energy trapped in the SSI model. Because the Subtraction Model does not impose the compatibility of displacements at the internal nodes within the excavated soil volume, it is reasonable to suspect that the energy entrapment occurs within the excavated soil model.

To better understand the deviation of the Subtraction Model's results from those of the target solution, the horizontal (x) and vertical (z) modes of the excavated soil model, restrained on the bottom and all four sides, are calculated using the FBASE module of MTR/**SASSI**. These modes, which correspond to the peaks of the horizontal and vertical transfer functions calculated at the top of the soil model from input

in the x- and z-directions, respectively, are shown in Figure 10. An examination of these modes shows some correlation to the observed peaks and valleys in the scattering and impedance functions calculated using the Subtraction Model (see **Error! Reference source not found.** 5 through Figure 9). By imposing the compatibility of displacements at the free-field surface nodes (i.e. the Modified Subtraction Model), these anomalies disappear, and the calculated results show good agreement with the target solution at all frequencies. One may suspect that this improvement is the result of shifting the modes of the soil model to frequencies beyond the frequency of interest by further restraining the top nodes. An examination of the modes of the soil model restrained on all sides, including the bottom and top (see **Error! Reference source not found.** 11), reveals a shift to somewhat higher frequencies while still remaining within the frequency range of interest. And because these modes have no effect on the results of the Modified Subtraction Model, it is difficult to imagine that they are solely responsible for the anomalies in the results of the Subtraction Model. Perhaps in this particular case, the generation of artificial surface waves at the free-field boundary at the top gives rise to additional wave reflections, which may result in energy being trapped within the soil model when the compatibility of displacements is not imposed at the surface nodes.

Conclusions

In general, the use of the Subtraction Model should be limited to cases where $a_o < 3$. For cases where $a_o > 3$, the use of the Subtraction Model may result in a significant number of erroneous peaks and valleys in the calculated transfer functions. The impact of these anomalies on the final results can be significant, particularly if they are affected by the energy of input motion.

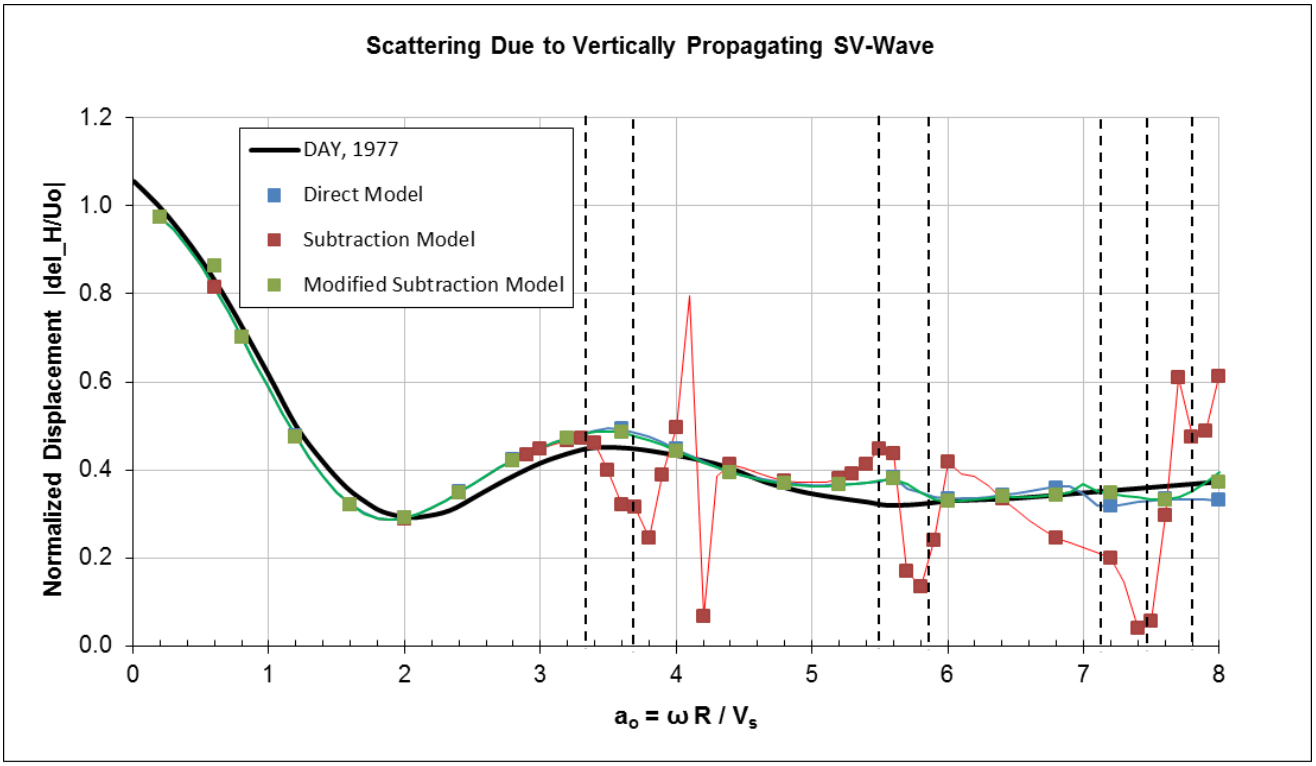
The Direct and Modified Subtraction Models show good agreement with published results, and are therefore superior to the Subtraction Model.

References

1. Day, S.M., “Finite Element Analysis of Seismic Scattering Problems,” Ph.D. Dissertation, University of California, San Diego, La Jolla, California, 1977.

2. Apsel, R.J. and Luco, J.E., “Impedance Functions for Foundations Embedded in a Layered Medium: An Integral Equation Approach,” Earthquake Engineering and Structural Mechanics, Vol. 15, No. 2, February, 1987.

3. MTR/**SASSI,** “System for Analysis of Soil-Structure Interaction,” Version 9.2, MTR& Associates, Inc., Lafayette, California, May, 2011.



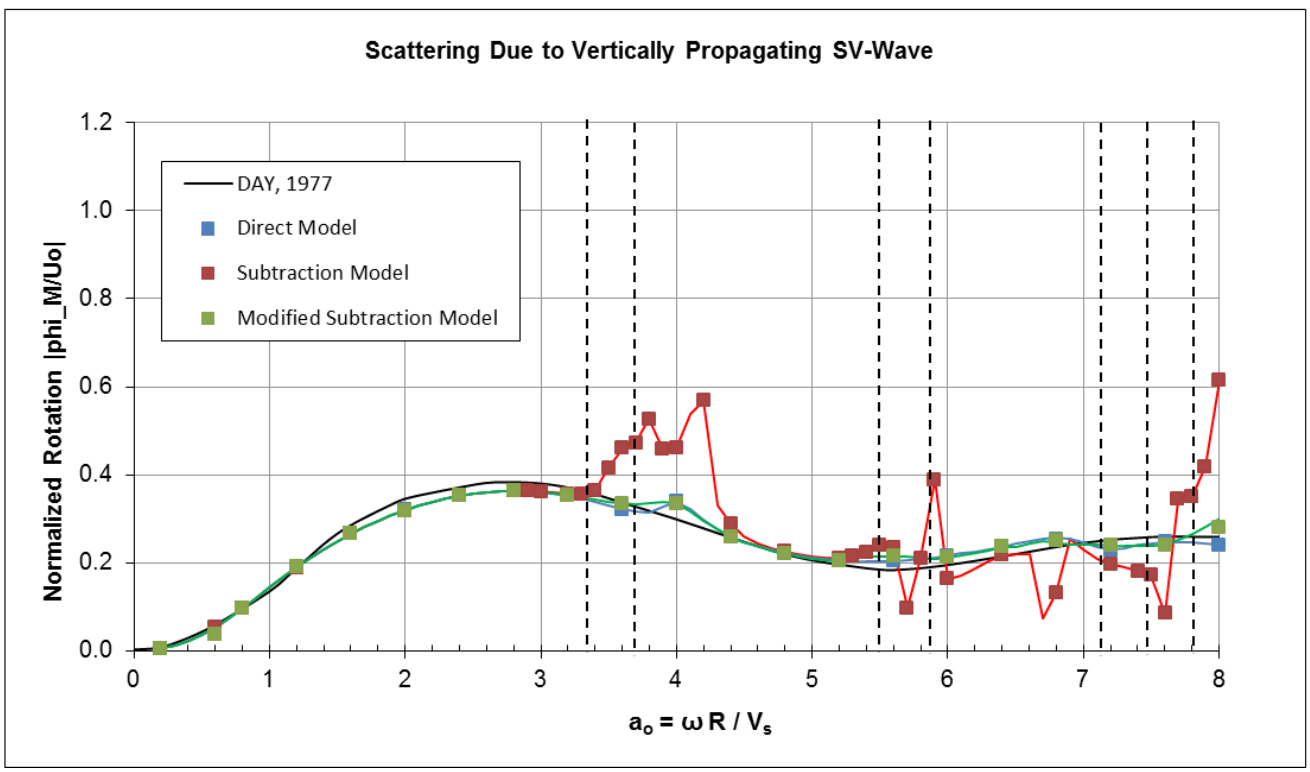
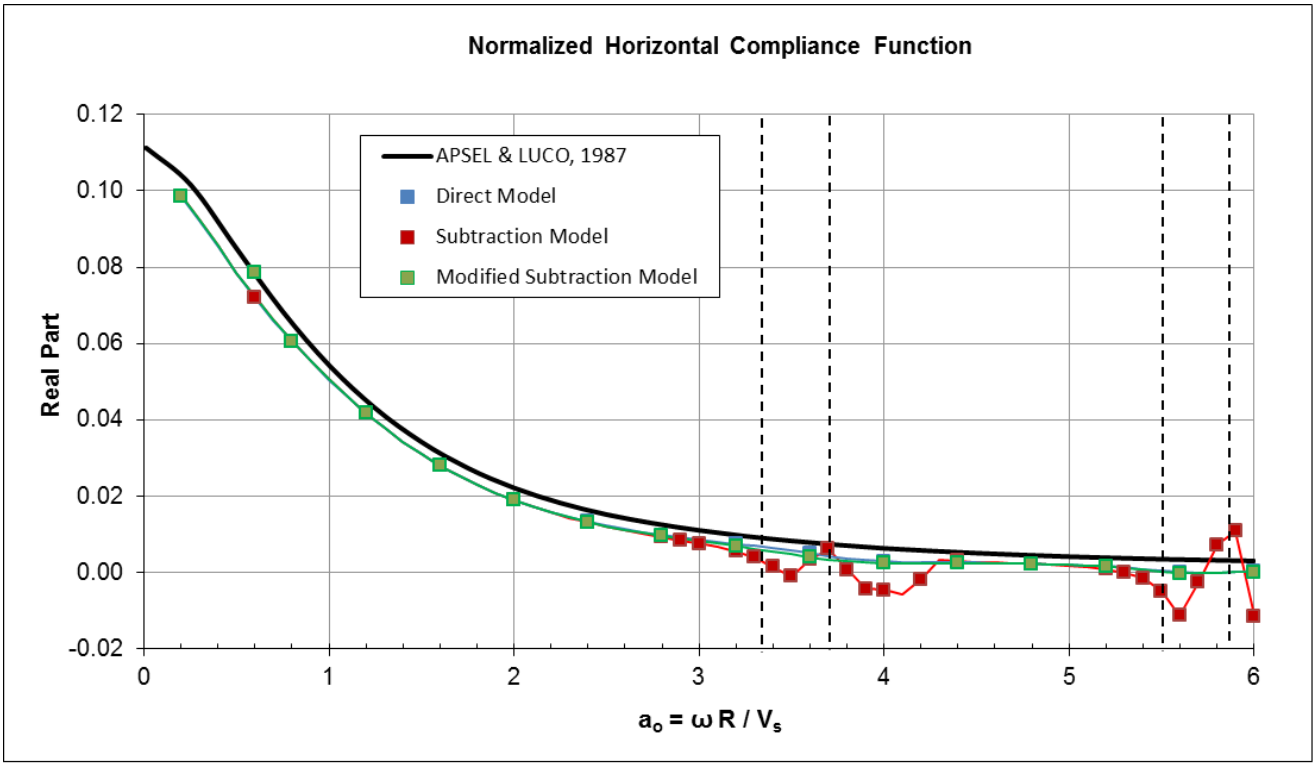


Figure 5: Comparison of Scattering Functions due to Vertically Propagating SV-Wave



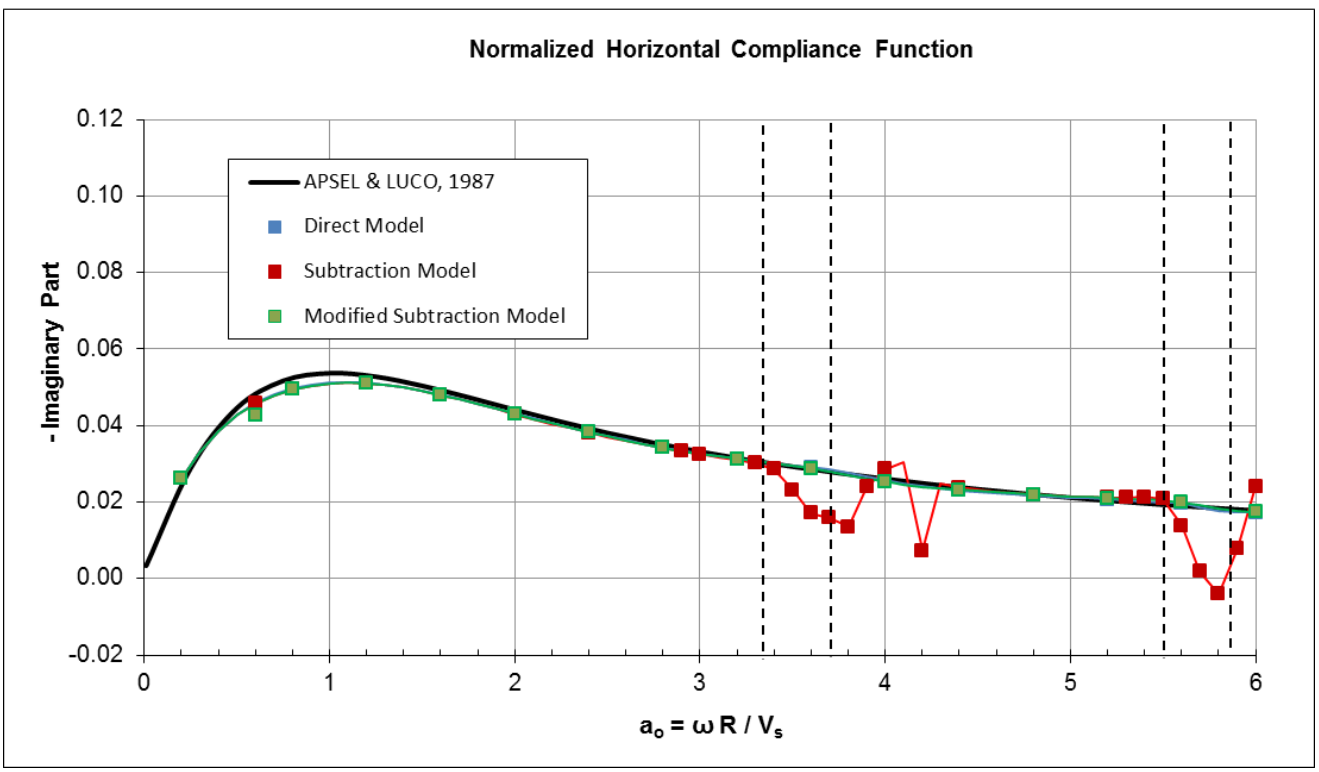
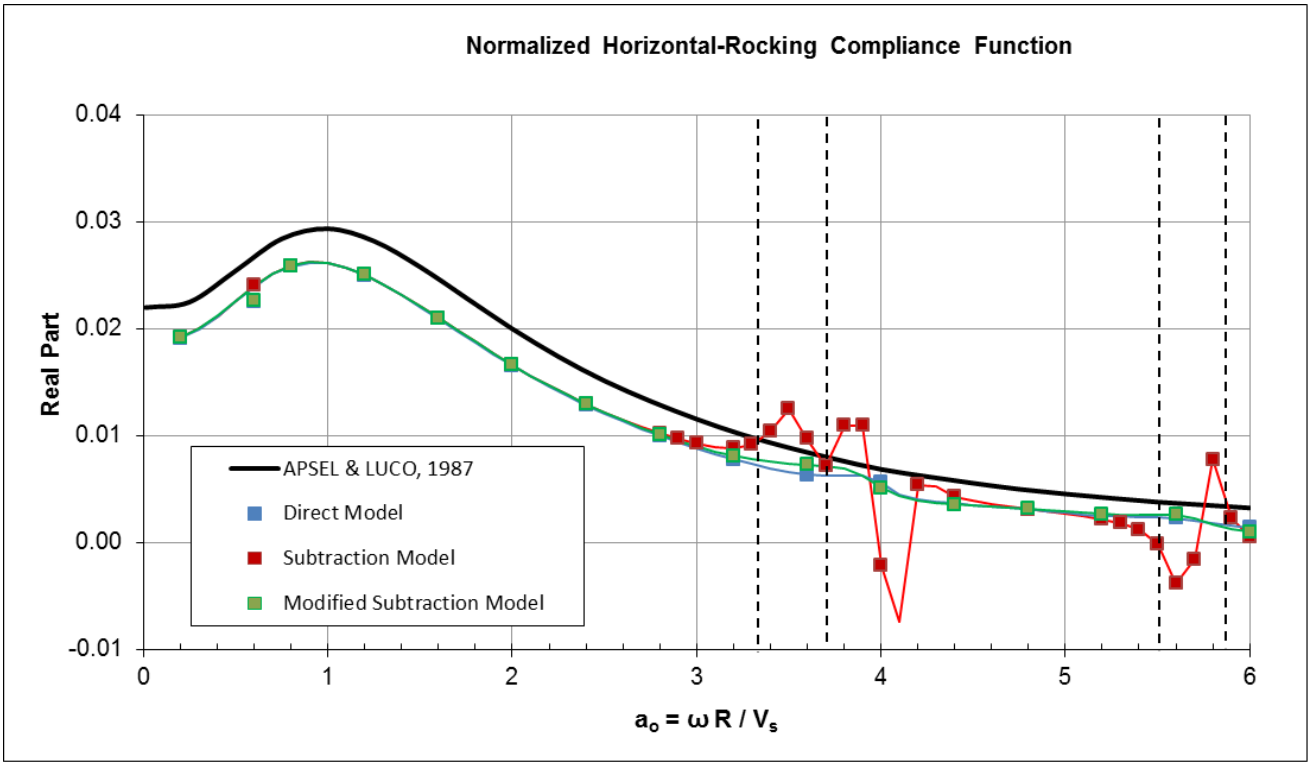


Figure 6: Comparison of Foundation Compliance Functions, Horizontal Component



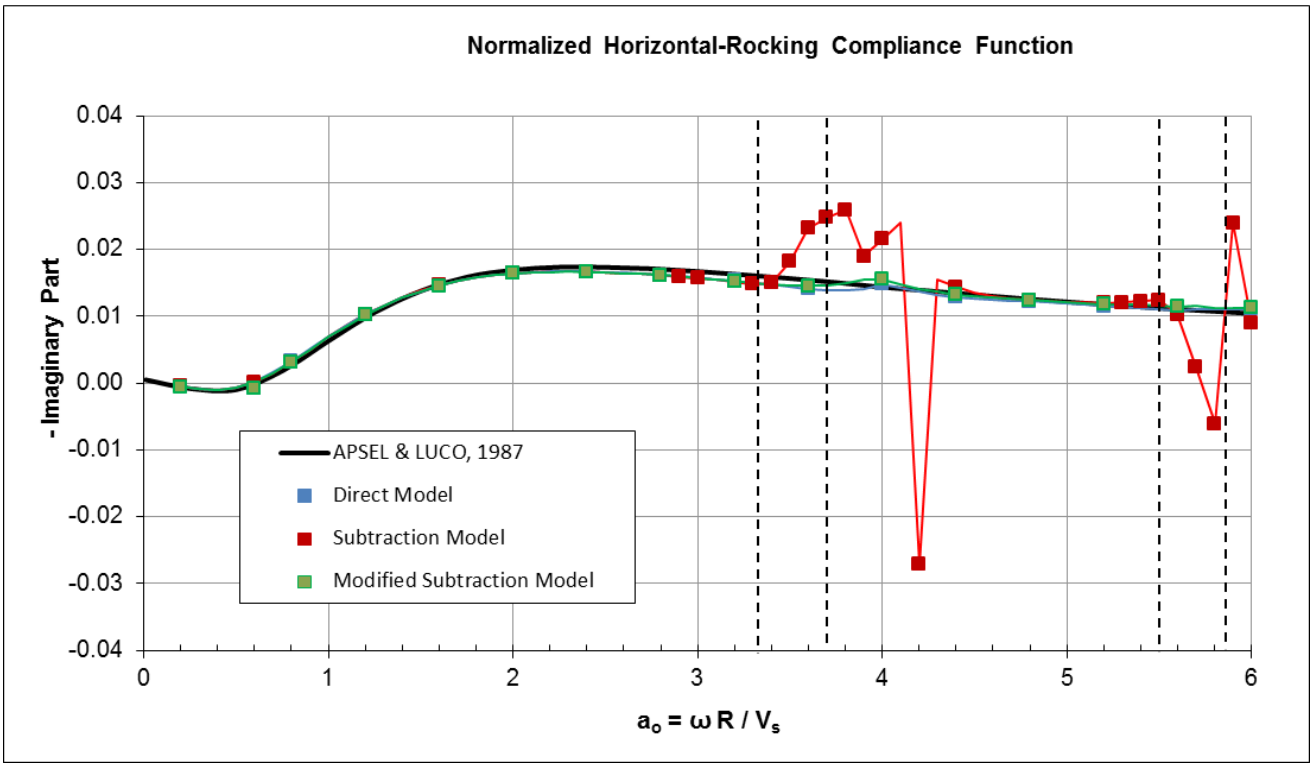
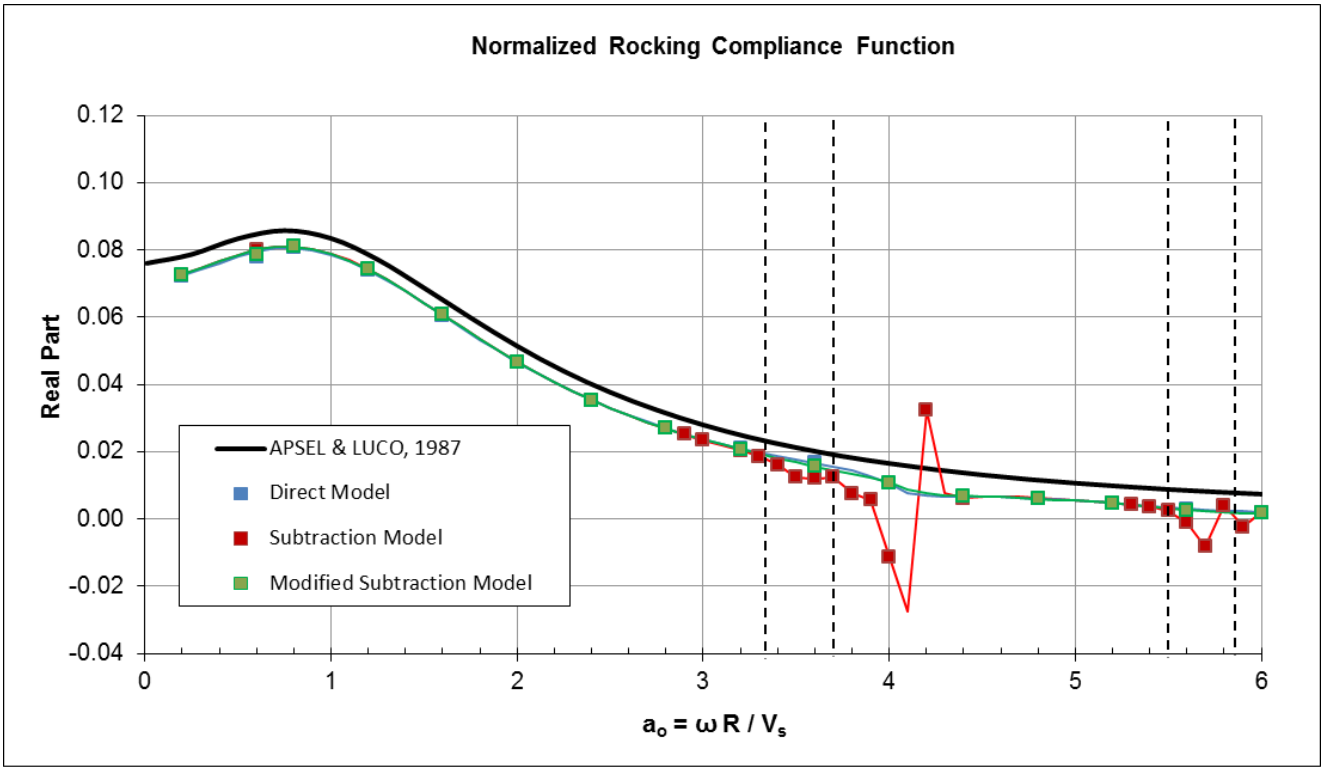


Figure 7: Comparison of Foundation Compliance Functions, Coupled Horizontal-Rocking Component



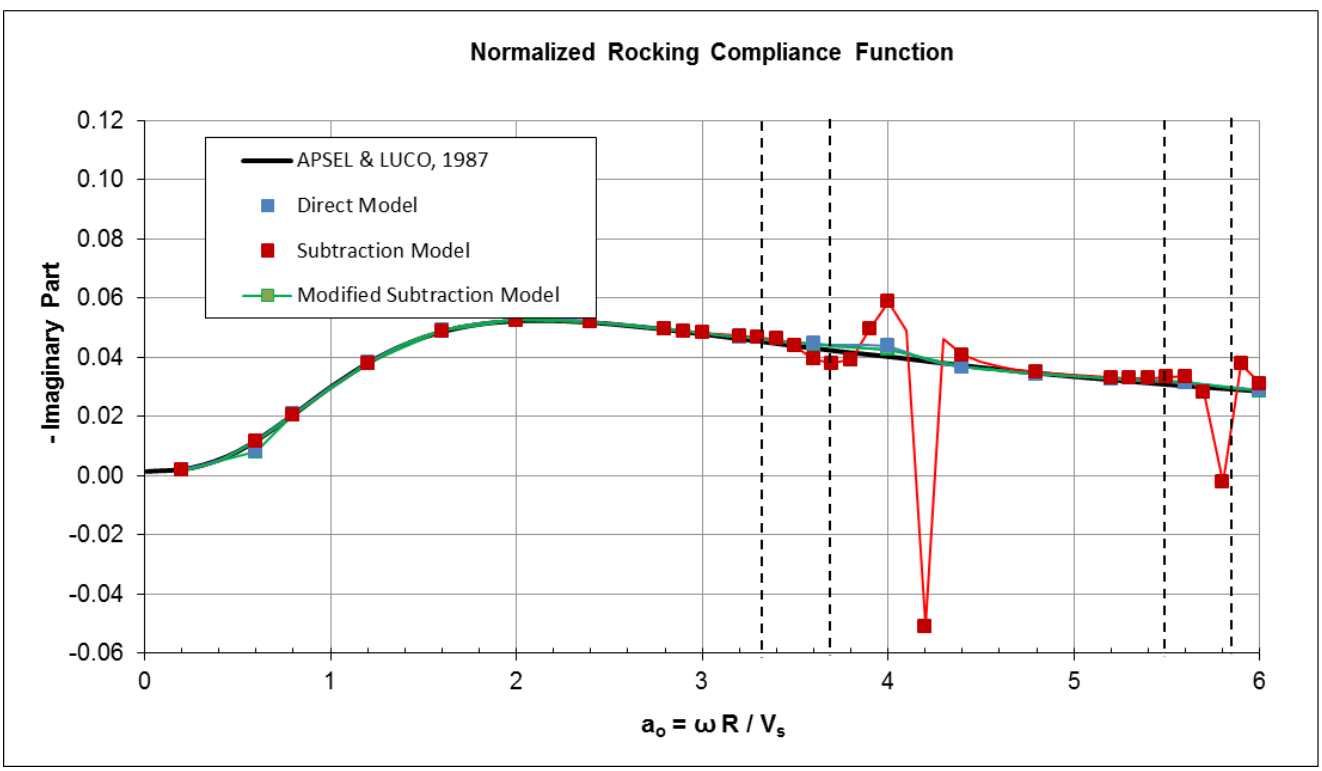
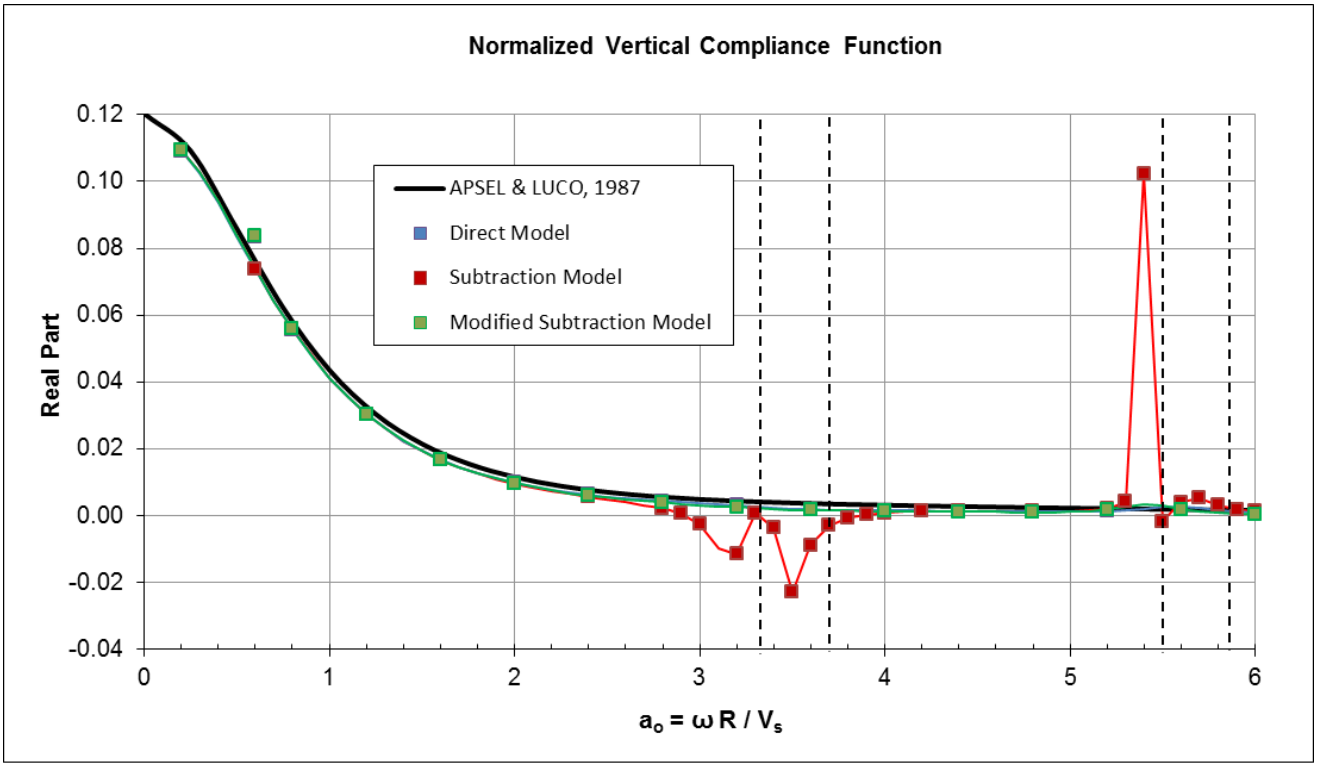


Figure 8: Comparison of Foundation Compliance Functions, Rocking Component



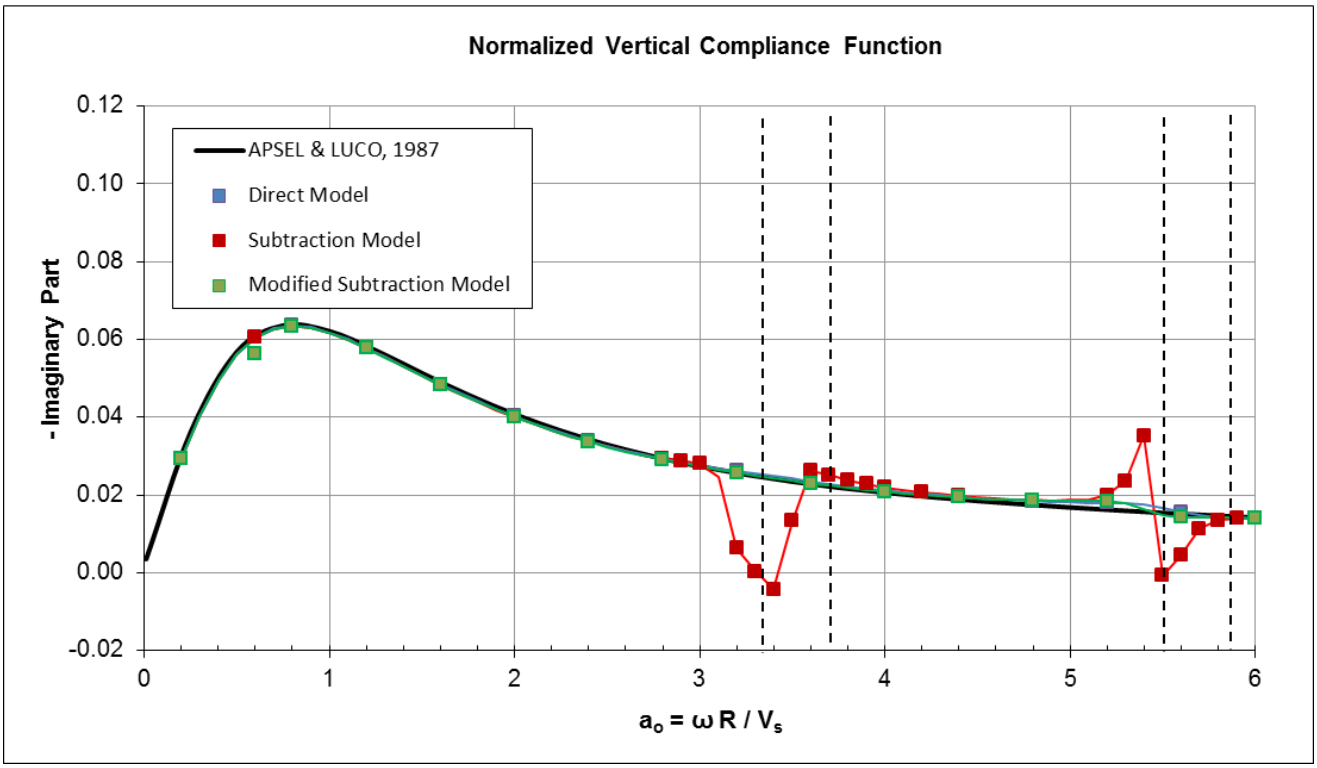


Figure 9: Comparison of Foundation Compliance Functions, Vertical Component

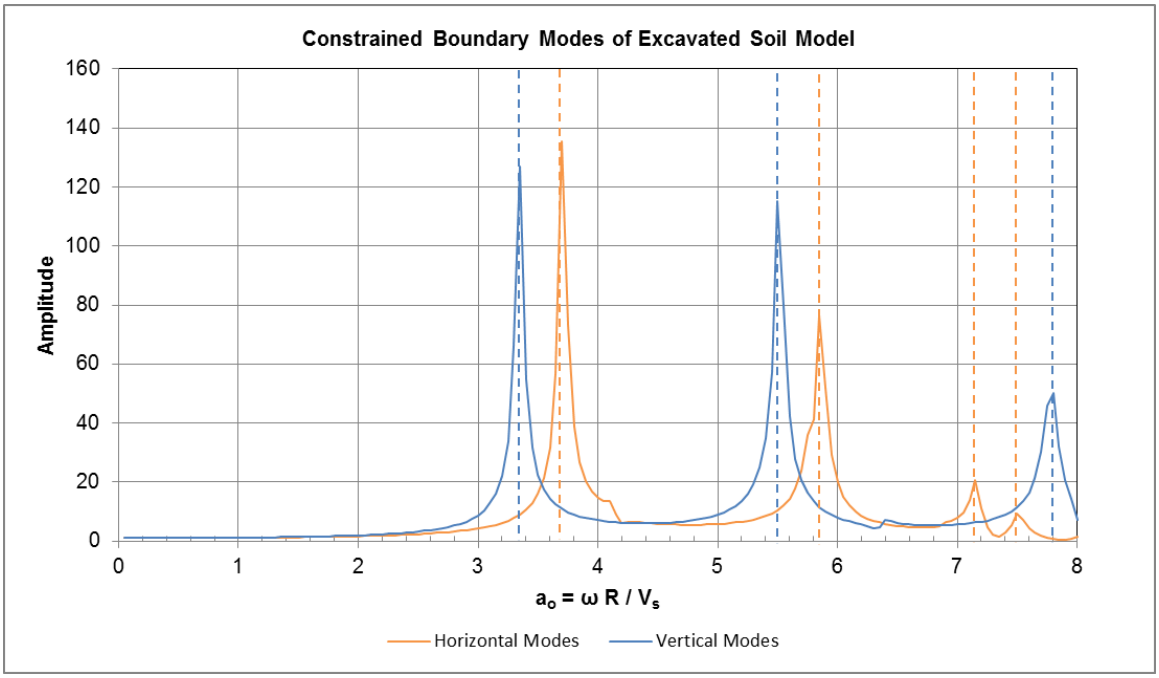


Figure 10: Horizontal and Vertical Modes of Excavated Soil Model Constrained on Four Sides plus Bottom Boundaries

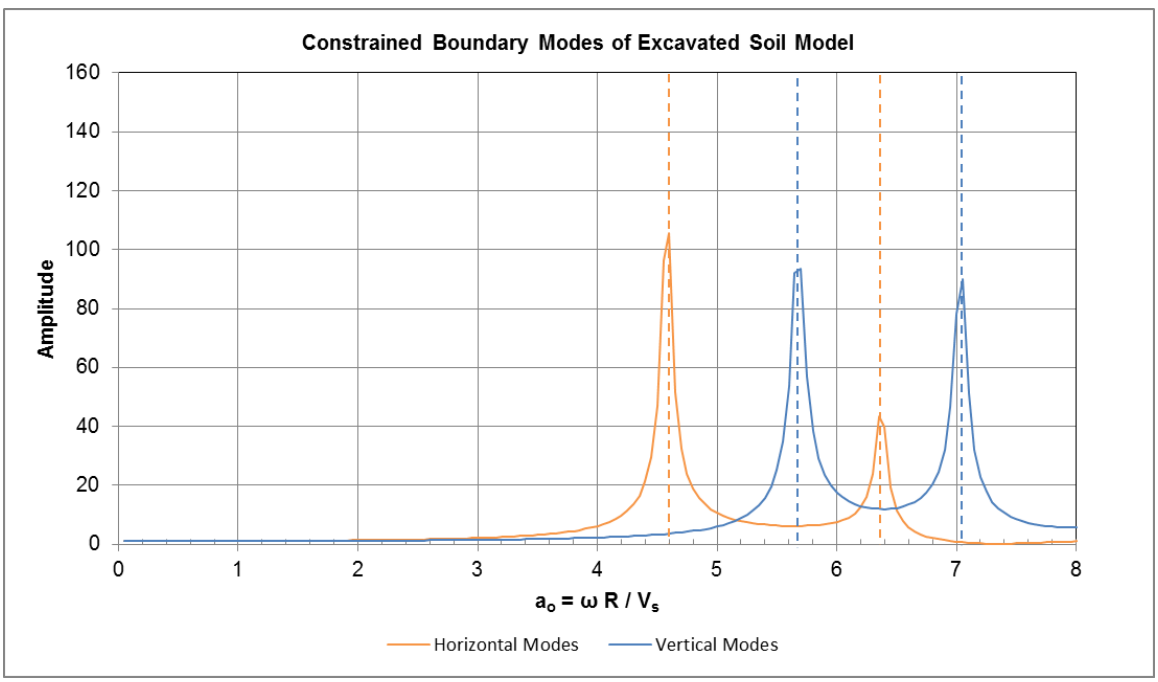


Figure 11: Horizontal and Vertical Modes of Excavated Soil Model Constrained on Four Sides plus Top and Bottom Boundaries