TECHNICAL NOTE

The Accuracy of Subtraction Method Examined in MTR/SASSI (Part 1)
June 20, 2011
MTR & Associates

In a recent letter to the Department of Energy (DOE) [1], the Defense Nuclear Facilities Safety Board (DNFSB) raised some concerns regarding the technical adequacy and proper validation of the Subtraction Method implemented in different versions of the SASSI program. The DNFSB reported that the response transfer functions calculated using the Subtraction Method for frequencies above 10 Hz exhibited peaks and valleys while those generated by the Direct Method were smooth and more reasonable. The DNFSB stated that the in-structure response spectra (ISRS) calculated using the Subtraction Method in some instances may become un-conservative for certain frequency ranges.

Purpose

The purpose of Part I of this document is to provide a brief background on several impedance modeling schemes used in SASSI [3], to examine the accuracy of three such schemes -- namely, the Direct, Subtraction and Modified Subtraction Models using MTR/**SASSI** [1] -- and to provide additional guidance in applying these models to large-scale SSI problems. The conclusions presented in this document are based on the results of two test problems examined as part of this study and included in Parts 2 and 3.

Theoretical Background

Direct Model

The SSI analysis methodology employed in SASSI, referred to as the Flexible Volume Method (FVM) [4, 5], is based on the observation that the solutions to scattering and impedance problems in the general sub-structuring approach can be greatly simplified if the interactions are considered over a volume rather than a boundary. In the FVM, the dynamic stiffness of the structure is reduced by the corresponding properties of the excavated soil volume, which is retained within the halfspace (i.e. horizontally layered). As a result of this, the scattering problem associated with a ground cavity reduces to that of the free-field ground response problem, while the impedance problem reduces to a point load solution in a horizontally layered system. The calculation of the impedance matrix thus involves performing an inversion of a full flexibility matrix associated with all flexible volume interaction nodes developed from a point load solution in a layered system. By imposing common degrees-of-freedom between the impedance model and the excavated soil model, the compatibility of the displacements at all interaction nodes -- including those within the excavated soil volume as shown in Figure 1 -- are satisfied. This ensures accurate and stable results that converge to the true solution as the mesh refinement is increased. This is the Direct Model (also referred to as “Direct Method”) of impedance calculation.

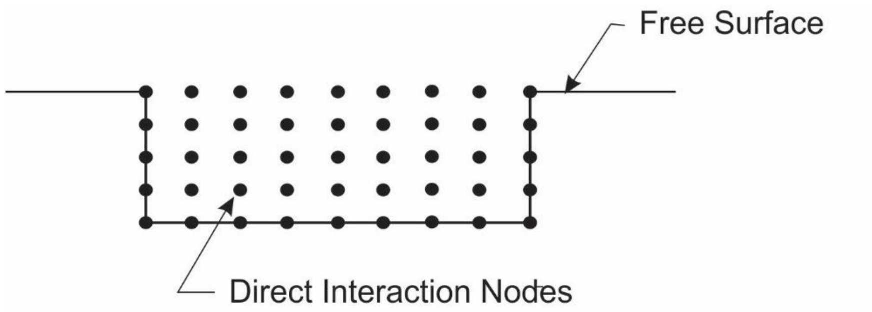

Figure 1 – Illustration of Direct Model

Skin Model

The Direct Model involves the inversion of a large, fully-populated, complex-valued flexibility matrix whose size grows approximately by the power of 3, as the dimensions and/or mesh refinement of the embedded foundation model increase in three-dimensional problems. To reduce the numerical effort involved in inverting a large flexibility matrix, the original SASSI program includes an alternative scheme for calculating the impedance matrix, referred to as the Skin Model (also called "Skin Method") [4]. In this model, only the degrees-of-freedom associated with the interaction nodes on the excavation skin (referred to as interface nodes) are considered in the inversion of the flexibility matrix, significantly reducing the numerical effort required in calculating the impedance matrix (see Figure 2 for a definition of the interface, intermediate and internal nodes).

In applying the Skin Model, it is not theoretically necessary to impose the compatibility of displacements at the internal nodes within the excavated soil volume. In reality, the internal nodes are fictitious and only included for mathematical convenience. The stiffness terms associated with the internal nodes are expected to cancel each other out when the dynamic stiffness of the excavated soil model is subtracted from the impedance matrix. The Skin Model imposes the compatibility of displacements at the interface nodes, but at the internal nodes this compatibility is only inferred. Because of the numerical difference in deriving the impedance matrix and dynamic stiffness of the excavated soil model, the Skin Model only provides acceptable impedance solutions if the cut-off frequency is set very low (i.e. to $V_s/12h$ or even lower, where V_s is the shear wave velocity of the foundation media and h is the smallest element size in the excavated soil model). As a result of this limitation, the Skin Model was never recommended for practical application. And as it has already been studied in detail [4] and remained largely unused, it will not be re-examined in this document.

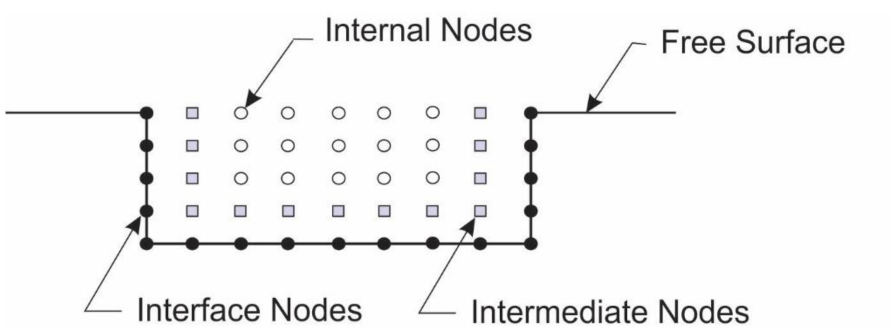

Figure 2 – Illustration of Skin Model

Symmetric Model

To further reduce the size of the impedance matrix, Symmetric and Anti-symmetric Impedance Models that take advantage of the system's symmetry were also developed and incorporated into the original SASSI program [4]. These models, derived from the special application of point loads in a layered system, significantly facilitated the SSI analysis of structures with large embedded foundations. But because the derivation of the Symmetric Models is exact and fully validated, they will not be re-examined in this document.

Rigid Model

Later attempts to further reduce the size of the impedance matrix led to the development of the Rigid (or constrained) Impedance Model [2]. This model is based on the assumption that the response of a rigid foundation can be fully described by 6 degrees-of-freedom (3 translations and 3 rotations). Taking advantage of the foundation's rigidity, the size of the complex-valued flexibility matrix was reduced to a 6 x 6 matrix, thus completely eliminating the need to invert a large impedance matrix. But because this feature is not available in the original SASSI program, as well as limited to foundations with rigid base slabs, it will not be discussed in this document.

Subtraction Model

The so-called Subtraction Model (also referred to as "Subtraction Method") is an alternative modeling scheme, later adopted by SASSI, for solving impedance problems. In this model, only the interface nodes are considered as interaction nodes, as shown in Figure 3 (i.e. the compatibility of displacements is no longer imposed at all interaction nodes within the soil volume). In some respects, this model is similar to the Skin Model, with one exception: the compatibility of displacements at the internal nodes is considered in the Skin Model, whereas in the Subtraction Model it is not imposed. The Subtraction Model gained popularity because, like the Skin Model, it significantly reduced the numerical effort involved in calculating the impedance matrix for large embedded structures. However, it suffers from the same issues of numerical accuracy that were originally observed in the Skin Model. These issues, as raised by the DNFSB, are further explored in this document.

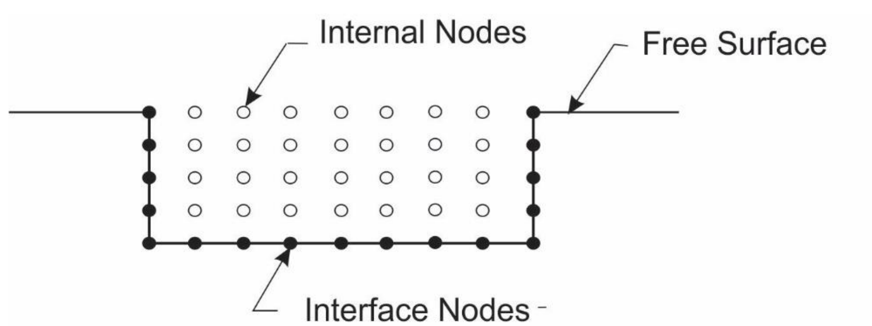

Figure 3 – Illustration of Subtraction Model

Modified Subtraction Model

The Modified Subtraction Model (also referred to as "Modified Subtraction or Enhanced Subtraction Method") is a proposed improvement over the Subtraction Model. According to this model, the compatibility of displacements – in addition to the skin nodes – is imposed at the internal nodes located on the free-field surface by specifying those nodes as interaction nodes (see Figure 4). The accuracy of the Modified Subtraction Model is further studied in this document.

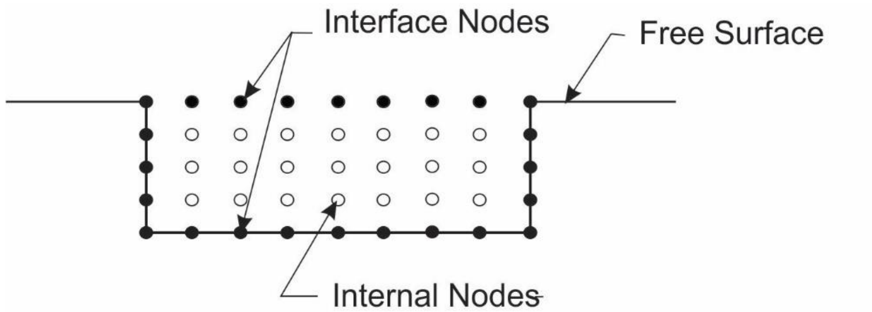

Figure 4 – Illustration of Modified Subtraction Model

MTR/SASSI Program

The MTR/**SASSI** program makes no distinction between the Direct, Skin, Subtraction, and Modified Subtraction Models, or any combination of the interaction node sets used to develop the impedance matrix. Because they are considered modeling schemes, the user simply specifies sets of interaction nodes (interface nodes) for which the compatibility of displacements is imposed. This set of interaction nodes is selected from amongst the excavated soil nodes, the balance of which is obtained automatically by the program and designated as "internal nodes". The compatibility of displacements is not imposed at the internal nodes.

The Direct Model, being the most accurate, specifies all the nodes within the excavated soil model as interaction nodes. The Subtraction Model, being the least accurate, specifies only the nodes on the excavation skin as interaction nodes. Other modeling schemes, such as the Skin and Modified Subtraction Models, specify more of the nodes within the excavated soil model as interaction nodes. In MTR/**SASSI** any impedance modeling scheme that does not impose the compatibility of displacements at all internal nodes (such as the Skin, Subtraction and Modified Subtraction Models) is actually a subset of the Direct Model with incompatible displacements.

Technical Issues of Subtraction Model

To investigate the potential technical issues raised by the DNFSB regarding the Subtraction Model, two test problems are included in MTR/**SASSI** to examine the accuracy of different impedance modeling schemes. The first is a benchmark problem that compares the results of the Direct, Subtraction and Modified Subtraction Models in terms of scattering and impedance solutions against those of published literature. The second represents a simplified model of a nuclear power plant (NPP) structure analyzed for a standard soil site in the Western United States (WUS) and a hard rock site in the Central and Eastern United States (CEUS). The results of the second model in terms of computed transfer functions, maximum accelerations, response spectra and dynamic soil pressures obtained from different modeling schemes are compared at several key locations in the structure. The details of the two test problems and a detailed discussion of the results are included as Part 1 and 2 of the technical note. A brief discussion of the results is provided below.

Based on an examination of the results provided in Part 1 and 2, the impedance and scattering solutions derived using the Subtraction Model are only found to be accurate up to $a_o = 3$, where $a_o = \omega R / V_s$ and

R is the equivalent foundation radius, V_s is shear wave velocity of soil media and ω is circular frequency. When the value of a_o exceeds 3, the computed response transfer functions exhibit erroneous peaks and valleys that are believed to be associated with the wave energy trapped within the excavated soil model. For the NPP model analyzed using the Subtraction Model, the transfer function departure occurs around 10-15 Hz for the standard soil site and 15-20 Hz for the hard rock site. The impact of the transfer function departure on the final results (such as maximum acceleration values, in-structure response spectra and dynamic soil pressures) are found to be significant at some locations in the structure.

When the compatibility of displacements is also imposed at the internal nodes located at the free surface (as in the Modified Subtraction Model), the transfer functions become smoother, and the erroneous peaks and valleys disappear for values of a_o up to about 8 (as they are examined in this document). The results of the Modified Subtraction Model are found to be closer to those of the Direct Model.

Conclusion

In general, the use of the Subtraction Model should be limited to cases where $a_o < 3$. For cases where $a_o > 3$, the Subtraction Model should be used with extreme caution as it may result in erroneous peaks and valleys in the calculated response transfer functions. The impact of these spurious modes on the final results can be significant, particularly if they are affected by the energy of input motion.

The results of the Modified Subtraction Model are close to those of the Direct Model, validated using a benchmark problem for a_o values up to about 8 and compared for a typical NPP model for both the standard soil and hard rock sites.

References

1. DNFSB, Washington DC, Letter to Deputy Secretary of Energy, "Issues Related to the SASSI Computer Software," Defense Nuclear Facilities Safety Board, Staff Issue Report, March 3, 2011.

2. MTR/**SASSI** [2011], "System for Analysis of Soil-Structure Interaction," Version 9.2, MTR & Associates, Inc., Lafayette, California, May.

3. Lysmer, J., Tabatabaie, M., Tajirian, F., Vahdani, S. and Ostadan, F. [1981], "SASSI – A System for Analysis of Soil Structure Interaction," Report No. UCB/GT/81-02, Geotechnical Engineering, Department of Civil Engineering, University of California, Berkeley, April.

4. Tabatabaie, M. [1982], "The Flexible Volume Method for Dynamic Soil-Structure Interaction Analysis." Ph.D. Dissertation, University of California, Berkeley.

5. Tajirian, F. [1981], "Impedance Matrices and Interpolation Techniques for 3-D Interaction Analysis by the Flexible Volume Method." Ph. D. Dissertation, University of California, Berkeley.