

## CURRENT METHODOLOGIES FOR INCORPORATING GROUND MOTION INCOHERENCY IN SSI ANALYSIS USING SASSI

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### ABSTRACT

Several deterministic methods are currently available for incorporating spatial incoherence of ground motions in the SSI analysis using SASSI. The Response SRSS method is the first implementation of the incoherent ground motions in the SASSI code that was done by EPRI in the late 90's. A decade later, this procedure was further investigated by EPRI for application to the licensing of the new reactors in the United States. From that study three other procedures -- Transfer Function SRSS (TF-SRSS), Algebraic Summation (AS) and Simulation Mean (SM) have been developed and recommended for applications to the new plant designs.

The EPRI AS and SM procedures are implemented at the ground motion level and TF-SRSS at the structural response level. The purpose of this paper is to present a brief overview of each of the above methodologies, and recommend two new procedures: TF-Summation and Response-Simulation implemented at the structural response level. A numerical possibility issue in the TF-SRSS procedure that can cause un-conservative results has been identified and discussed.

The seismic response of a reduced nuclear island stick model supported at the ground surface and subjected to three orthogonal components of spatially incoherent ground motions are calculated using the TF-SRSS and new TF-Summation and Response-Simulation procedures implemented in MTR/SASSI. The results are then compared against each other and those reported by EPRI using the stochastic method in CLASSI. Conclusions and recommendations regarding the accuracy of each method and their implementation scheme are provided.

### INTRODUCTION

Seismic soil-structure interaction (SSI) analysis of structures subjected to incoherent ground motions involves two main aspects – characterization of the free-field input motion at foundation interaction nodes and implementation of the incoherency effects in the SSI analysis.

#### *Characterization of Incoherent Ground Motion*

Several empirical coherency models to characterize ground motion coherency on a horizontal plane have been developed and discussed in Abrahamson (2006) and EPRI (2007a). These models are primarily based on statistical analysis of the ground motions recorded on soil or rock sites from dense arrays. Because of the random nature of these motions, their spatial variations are best characterized statistically by coherency functions that relate the coherency (or similarity) of the motions at two adjacent stations in terms of the station spacing and frequency of the motion. The coherency function is then anchored to a reference station to describe the seismic wave field. Figure 1 shows typical horizontal and vertical coherency functions for rock (EPRI 2007b), which are also used in the current study. As seen from Figure 1, the ground motion coherency decreases with increasing station spacing and frequency. The coherency functions are generally expressed in terms of mathematical functions for ease of implementation in engineering applications. The coherency functions for the horizontal and vertical components of ground motion are uncoupled.

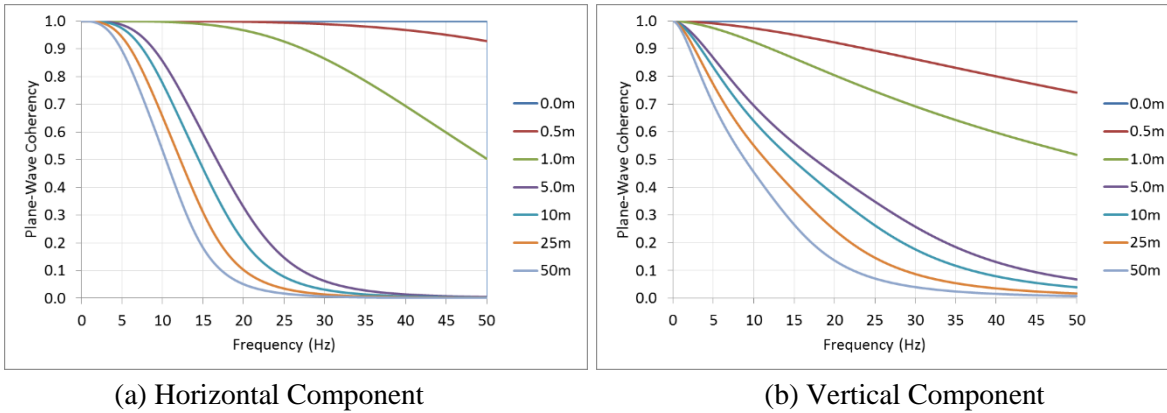


Figure 1. Plane-Wave Coherency Functions

### *Implementation of Incoherency Effects in SSI Analysis*

Both probabilistic and deterministic methods have been used to implement the effects of ground motion incoherency in seismic response of structures. Although simplified methods had been used to explain and account for the effects of ground motion incoherency, a more systematic approach that considered site-specific effects and utilized actual empirical coherency data was first investigated by Tseng and Lilhanand, 1997 for the Diablo Canyon Long Term Seismic Program (DCLTSP). The DCLTSP study describes in detail the mathematical basis for implementing the effects of seismic wave incoherency in the SSI analysis using probabilistic (CLASSI) and deterministic (SASSI) approach. For SASSI implementation, The Response SRSS method was used. Validation of analysis methodology and computer programs were presented together with comparison of the analytical results with experimental data. Recent studies to address the new plant seismic issues resolution by EPRI (2007b) expanded on the DCLTSP study with further investigation and validation of ground motion incoherency effects and its implementation in the SSI analysis using CLASSI and SASSI codes. The new implementations of incoherency effects in SASSI, included the Transfer Function SRSS (TF-SRSS), Simulation Mean (SM) and Algebraic Summation (AS) methods. These methods allow direct output of the response time histories that are often required for evaluation of subsystems, not provided by the Response SRSS method used for DCLTSP.

Implementation of the ground motion incoherency in the SSI analysis using SASSI is the main focus of this paper. A brief overview of the key formulations essential for understanding the different implementation schemes are presented. This includes the currently published methods (Response SRSS, TF-SRSS, Algebraic Summation and Simulation Mean) as well as two new alternative methods (Transfer Function Summation and Response Simulation) presented herein.

### **OVERVIEW OF COHERENCY IMPLEMENTATIONS IN SASSI**

Implementation of the ground motion incoherency in SASSI is based on the spectral decomposition of coherency matrix of the stochastic seismic wave field characterized by the coherency functions. Assuming that the ground motions are stationary random process and have the same power spectral density (PSD) function over the entire foundation region as that of the reference station, the cross spectral density (CSD) function for two separate locations  $j$  and  $k$  at the free-field ground surface for each horizontal and vertical component of the reference station can be expressed in the frequency domain as follows:

$$S_{jk}(\omega) = [S_{jj}(\omega) \cdot S_{kk}(\omega)]^{1/2} \cdot \gamma_{jk}(\omega) \quad (1)$$

Where  $S_{jj}(\omega)$  and  $S_{kk}(\omega)$  are the PSD of the seismic motion at locations  $j$  and  $k$ , and  $Y_{jk}(\omega)$  is the plane-wave coherency between locations  $j$  and  $k$ , and  $\omega$  is the circular frequency. In general,  $Y_{jk}(\omega)$  is complex-valued and referred to as unlagged coherency. For this formulation, we have used lagged coherency model that separates the stochastic part or randomness of the ground motion amplitude from the deterministic part (i.e. wave-passage effects) so each can be considered separately. The lagged coherency is real-valued and positive. Based on Eq. 1, the coherency matrix  $S^{s,i}_{jk}$  for all interaction nodes of the foundation can be readily constructed for any frequency of input motion. In the following formulations, “g” and “s” refer to ground and structure motions, and “c” and “i” refer to coherent and incoherent components of motion, respectively.

Expressing the ground motion,  $U^g_k(\omega)$  in terms of the coherent (deterministic) and incoherent (stochastic) components at each interaction DOF  $k$ , one can write:

$$U^g_k(\omega) = H^{s,i}_k(\omega) \cdot H^{s,c}_k(\omega) \cdot U^g_o(\omega) \quad (2)$$

Where  $H^{s,c}_k(\omega)$  and  $H^{s,i}_k(\omega)$  are the coherent and incoherent components of the free-field motion, respectively, and  $U^g_o(\omega)$  is the reference motion. The coherent component of motion can be obtained from deterministic analysis of plane-wave propagation in SASSI, which accounts for the wave-passage effects. The incoherent component of motion can be represented in terms of the Eigen properties of the coherency matrix based on the covariance factorization technique as expressed below:

$$H^{s,i}_k(\omega) = \sum [\varphi_{m,k}(\omega) \cdot \lambda_m(\omega) \cdot \eta_{\theta m}(\omega)] \quad m = 1, M \quad (3)$$

Where  $\lambda_m$  and  $\varphi_{m,k}$  are the  $m$ -th eigenvalue and eigenvector component of coherency matrix at interaction node  $k$  and  $\eta_{\theta m}(\omega) = \exp(i\theta_m)$  is the random phase factor associated with the  $m$ -th eigenmode. The summation  $\sum$  is over the number of eigenmodes,  $M$ .

By considering that the structural response at any degree of freedom  $j$ ,  $U^s_j(\omega)$  consists of the superposition of the effects produced by application of the ground motion at each interaction DOF  $k$ , one can write:

$$U^s_j(\omega) = \sum \{ H^{s,j,k}(\omega) \cdot \sum [\varphi_{m,k}(\omega) \cdot \lambda_m(\omega) \cdot \eta_{\theta m}(\omega)] \cdot H^{s,c}_k(\omega) \} \cdot U^g_o(\omega) \quad m = 1, M \ \& \ k = 1, N \quad (4)$$

Where  $H^{s,j,k}(\omega)$  is the complex-valued transfer function for structural DOF  $j$ . The outside summation  $\sum$  is over  $N$ , where  $N$  is the number of interaction DOF's.

The only unknown in Eq. 4 is the random phase angle,  $\theta_m$ , which is taken as uniformly distributed between  $-\Pi$  and  $+\Pi$  with mean value of 0. Characterization of the random phase angle forms the basis for solution of Eq. 4 in different implementations of coherency effects in SASSI. The following provides a brief overview of the currently available coherency implementation methods in SASSI. The numerical concerns with the TF-SRSS, AS and SM methods are discussed and two new alternative implementations of incoherency in SASSI -- the TF-Summation and Response-Simulation Methods with the aim of addressing these numerical concerns are presented.

It is noted that all implementations are done in the frequency domain, whereby the response of the structure is calculated for a selected number of discrete frequencies of the Fourier transform of control motion with the results interpolated to construct the full transfer function. The resulting complex-valued transfer function at each DOF of the structure is then used to calculate the response of the structure. Although the solution for each frequency is independent of other frequencies, the final results can be significantly affected by the interpolation scheme depending on the number and accuracy of the computed frequencies. This is an important factor for obtaining reasonably accurate results using each of the methods discussed below.

### ***Response SRSS Method***

In this implementation, the structural response due to incoherent ground motion associated with each mode  $m$  of the coherency matrix is first determined without including its random phase angle (i.e.  $\eta_{\theta m}(\omega) = 1$ ). This results in the following equation, which is then solved for each frequency of analysis.

$$U^{s,m_j}(\omega) = \sum [H^{s,m}_{j,k}(\omega) \cdot \varphi_{m,k}(\omega) \cdot \lambda_m(\omega) \cdot H^{g,c}_k(\omega)] \cdot U^{g_o}(\omega) \quad k = 1, N \quad (5)$$

Where  $H^{s,m}_{j,k}(\omega)$  and  $U^{s,m_j}(\omega)$  are the transfer function component and structural response associated with mode  $m$  of the coherency matrix. The time history response is then obtained for each mode from convolution with the reference motion. Following this, the maximum response of interest at any structural DOF  $j$  is computed from combining the contribution of responses of all dominant modes by taking the SRSS of the maximum responses resulting from individual modes. The numerical effort for this method is controlled by the number of significant modes to be considered in the solution.

### ***Transfer Function SRSS Method***

The transfer function SRSS procedure (TF-SRSS) is described in EPRI (2007b). In this implementation, similar to Response SRSS method, Eq. 5 is used to calculate the structural response due to incoherent ground motion associated with each mode of the coherency matrix without including its random phase angle. Then for each frequency, the contribution of the effects of individual modes to the transfer function at any structural DOF  $j$  is calculated by taking SRSS of the transfer function from individual modes, as shown in Eq. 6 below.

$$U^s_j(\omega) = \text{SQRT}[\sum \{[U^{s,m_j}(\omega)]^2\}] \quad m = 1, M \quad (6)$$

The resulting transfer function at any given structural DOF  $j$ , which now includes the contribution of all the significant modes is convolved with the reference motion to calculate time history of the response.

The numerical effort for this method is also controlled by the number of significant modes to be solved, and therefore, comparable to that of the Response SRSS method.

### ***Algebraic Summation Method***

The Algebraic Summation (AS) procedure is discussed in EPRI (2007b). Based on a presumption that ‘‘Median Input produces Median Response’’, spectral factorization of the coherency matrix with mean phase angle for each spectral mode (i.e.  $\theta_m = 0$ ) is used to calculate the incoherent motion at the ground motion level. The ground motion,  $U^{g_k}(\omega)$  at any interaction node  $k$  including wave passage effects is calculated from Eq. 7 after algebraically summing the incoherent component of transfer function for all the significant modes  $m$  of the coherency matrix using square root of eigenvalues.

$$U^{g_k}(\omega) = \sum [\varphi_{m,k}(\omega) \cdot \lambda_m(\omega)^{1/2}] \cdot H^{g,c}_k(\omega) \cdot U^{g_o}(\omega) \quad m = 1, M \quad (7)$$

The structural response at any DOF  $j$  is then calculated by adding the effects of input motion at all interaction nodes,  $N$  using Eq. 8 below and convolving the transfer function with the reference motion.

$$U^s_j(\omega) = \sum [H^{s,j,k}(\omega) \cdot U^{g_k}(\omega)] \quad k = 1, N \quad (8)$$

The numerical effort for this method is small because only one complete solution is required as for the case of coherent input motion. This is a fast and quick analysis of incoherency in SASSI.

### ***Simulation Mean Method***

The Simulation Mean (SM) procedure is also described in EPRI (2007b). The implementation of this method is very similar to the Algebraic Summation except that the incoherent ground motion is obtained by the simulation process through phase angle randomization. In other words, for each mode of the coherency matrix a random phase angle is sampled between  $-\Pi$  and  $+\Pi$  and used to solve Eq. 9 to obtain the incoherent ground motion including the effects of wave passage at any interaction node  $k$ .

$$U_k^g(\omega) = \sum [\varphi_{m,k}(\omega) \cdot \lambda_m(\omega) \cdot \eta_{\theta m}(\omega)] \cdot H_k^{g,c}(\omega) \cdot U_o^g(\omega) \quad m = 1, M \quad (9)$$

The structural response at any DOF  $j$  for each simulation is then calculated by adding the contribution of input motion from all interaction nodes,  $N$  using Eq. 8 above and convolving the final transfer function with the reference motion. The final response is then calculated as the mean of the response from all simulations (i.e. solution with a separate set of randomized phase angle). This procedure is basically a repeat of Algebraic Summation for the number of simulations performed. Because each simulation is a new analysis, the numerical effort of this method is controlled by the number of simulations required to obtain convergent results.

### **ISSUES WITH CURRENT METHODS**

There are some issues with currently available incoherency methods that make them unattractive for implementation and/or use. These issues are discussed below.

Response-SRSS: The Response-SRSS Method does not generate time histories of response that may be needed to evaluate equipment or secondary systems. Therefore, it is not attractive for general applications.

TF-SRSS: Recent studies of ground motion incoherency in SASSI have revealed a numerical issue with this method. This numerical issue, further discussed below, appears to cause inaccurate and un-conservative incoherent responses in the structure.

AS and SM: As mentioned before, the AS and SM Methods combine the effects of all spectral modes at the ground motion level by assuming either a mean phase angle of zero or random phase angles. In either case, the results are the same: transfer functions that exhibit numerous spurious peaks from interpolation [EPRI, 2007b]. To address this problem, a large number of frequency solutions as well as numerical smoothing and conditioning of the transfer functions are required to minimize the effects of spurious peaks. This becomes a difficult task especially when dealing with detailed FE models.

### **NUMERICAL POSSIBILITY IN TF-SRSS METHOD**

Assume for one moment that we are dealing with only a single mode in the solution, in which case one would expect to obtain the same solution from the TF-STRSS Method regardless of whether SRSS was performed or not. But in fact the square root of a number has two possible values, positive and negative. Since a positive or negative sign can affect the outcome, the question becomes which sign do we use? When dealing with a single mode, it is possible to select the sign of the outcome of the square root for each frequency solution such that it is aligned with the sign of the pre-squared value to avoid this numerical issue. But it is not possible to do the same when more than one mode is involved. In other words, retaining the correct sign of the combined eigenmodes is important, and it does not seem to be doable when SRSS is performed on more than one mode, as in the TF-SRSS Method. As a side note, the use of SRSS in the Response SRSS Method is valid because it is applied at the final response level for each spectral mode (similar to modal superposition time history analysis) rather than at the transfer function level. As a result, the orthogonality of the mode shapes remains valid.

## NEW ALTERNATIVE PROCEDURES

To address the above issues, two new alternative methods are presented below.

### *Transfer Function Summation Method*

The Transfer Function Summation Method was developed to address the numerical issue associated with the TF-SRSS Method. The new TF-Summation Method is similar to TF-SRSS except, after solving for each significant mode of the coherency matrix with a phase angle of zero, the contribution of the effects of the individual modes at any DOF in the structure is calculated by algebraic summation rather than taking the SRSS of the transfer functions from individual modes. It reflects the mean input while eliminating the possibility of more than one numerical outcome, as shown in Eq. 10 below.

$$U_j^s(\omega) = \sum [U_j^{s,m}(\omega)] \quad m = 1, M \quad (10)$$

### *Response Simulation Method*

This new procedure basically repeats the Transfer Function Summation Method for multiple simulations, each using a random phase angle rather than a phase angle of zero for each significant mode of the coherency matrix. The final result is then obtained by taking the mean response of all the simulations.

The newly implemented TF-Summation and Response-Simulation Methods have a key advantage. The calculated transfer functions from each simulation (with TF-Summation being just one simulation with a zero phase angle) are not affected by spurious peaks from interpolation because the spectral mode shapes of the coherency matrix are better suited for individual input to the SSI model rather than being combined into one input at the ground motion level, as is the case with the AS and SM procedures.

The TF-Summation and Response-Simulation Methods require the same numerical effort as the TF-SRSS Method controlled by the number of significant modes to be solved. Response-Simulation does not require any significant additional effort because each simulation is part of the post-processing effort.

## COMPARISON OF COHERENCY METHODS

To evaluate the accuracy and effectiveness of TF-SRSS, TF-Summation and Response-Simulation procedures, as discussed above, a similar test problem cited in EPRI (2007b) was used. It is noted that in this comparison all methods are implemented in the same program (MTR/SASSI), use the same transfer function results at the structural response level solved for the same number of frequencies and interpolated in the same fashion without any further adjustment and/or conditioning of the transfer functions.

### *Structure and Foundation Models*

Figure 2 shows the idealized stick model of a nuclear island with rigid foundation founded on hard rock. The structure consists of three concentric sticks with some interconnectivity representing the auxiliary/shield building (ASB), Steel Containment Vessel (SCV) and Containment Internal Structures (CIS). No torsion due to mass offset is considered. The foundation base slab is 150 x 150 ft. in plan dimensions. The rock velocity profile is shown in Figure 3. The ASB has predominant frequencies less than 10 Hz. The predominant modes of the SCV are 5.5, 6.14 and 16 Hz for the X-, Y- and Z-directions. The predominant frequencies of the CIS are higher than 10 Hz.

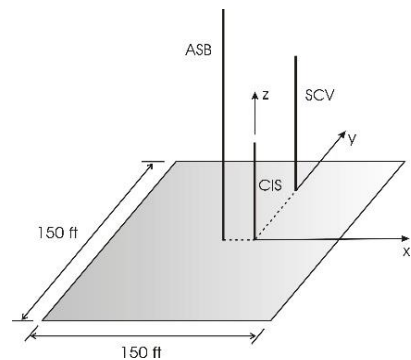


Figure 2. Idealized Model

## ***Input Motions***

The input motion consists of three orthogonal components of acceleration time histories spectrally-matched to uniform hazard rock spectra containing high frequency content. Figure 4 shows comparisons of the 5%-damped acceleration response spectra of input time histories with target spectra for the three components of the input motion. The control motion is applied at the free-field ground surface assuming vertically propagating plane waves (i.e. no wave passage effects).

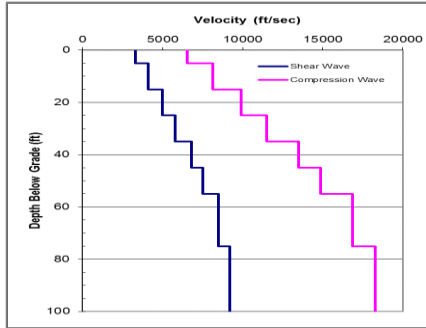


Figure 3. Velocity Profile

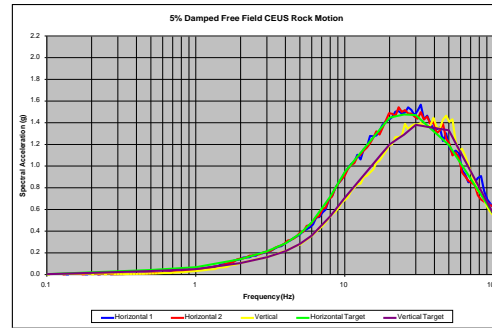


Figure 4. Comparison of Input vs. Target Spectra

## ***SSI Analysis***

The computer program MTR/SASSI (Tabatabaie, 2014) was used to obtain the response of the structure due to spatially incoherent ground motions. The horizontal and vertical rock coherency functions shown in Figure 1 are used for this study. The TF-SRSS, TF-Summation and Response-Simulation procedures, as discussed above were used to calculate and compare the incoherent response of the structure.

## ***Comparison of Incoherent Responses***

The effect of number of significant spectral modes on the calculated responses was examined for each method by comparing the typical structural responses calculated at the tops of ASB and SCV sticks considering 5, 10 and 20 modes of the coherency matrix. The results in terms of 5%-damped acceleration response spectra in the y- and z-directions are shown in Figure 5 for TF-SRSS, Figure 6 for TF-Summation and Figure 7 for Response-Simulation. Also shown in these figures are the corresponding target results reported from probabilistic analysis using CLASSI (EPRI 2007b). As shown in Figures 5 through 7, for the stick model of nuclear island considered, all three methods give similar results with 5, 10 and 20 significant modes indicating that a convergent solution is obtained for all three methods with a minimum of 5 modes. The results for the Response-Simulation represent the mean of 20 simulations.

The effect of number of simulations used in the Response-Simulation method to achieve a convergent solution was investigated by comparing the spectra at the same locations above from analysis with 5, 10 and 20 simulations with 10 significant modes. From the results shown in Figure 8 and inspection of the results at other locations not shown here, we find that using 10 simulations would be sufficient to obtain convergent solutions from the Response-Simulation method for the structures considered in this study.

Finally, the adequacy of the above procedures implemented in MTR/SASSI was examined by comparing the results of each method against those of the target results from CLASSI.

**TF-SRSS:** Figure 5 shows comparison of TF-SRSS results versus target results from CLASSI at the same locations discussed above. As shown in Figure 5 the results show significant difference between the two

results with TF-SRSS in general providing lower results both in terms of spectral amplitudes and zero-period accelerations. The cause of this numerical disparity was discussed above in this paper.

TF-Summation: The comparison of the results with CLASSI is shown in Figure 6. The overall shape of the calculated spectra shows reasonably good agreement with the CLASSI results. However, the spectral amplitude seems to be somewhat lower than the target values at frequencies above 8Hz and zero-period acceleration is slightly lower than the target values. The difference between the two results is attributed to the simplified assumption associated with zero phase angle in the modal combination.

Response-Summation: Figure 7 and 8 show comparison of the results with CLASSI. An inspection of the results show excellent agreement with the CLASSI results both in terms of the spectral amplitudes and zero-period accelerations at all locations. These results are typical at other locations.

## CONCLUSION

- The accuracy of TF-SRSS, TF-Summation and Response-Simulation procedures to incorporate the effects of incoherency in SASSI is assessed against probabilistic approach in CLASSI.
- For typical structural stick models, convergent results can be obtained using 5 dominant modes with all three methods and 10 simulations in Response-Summation method.
- TF-SRSS results in general are un-conservative and can deviate significantly from actual solution because of numerical possibility issue with the SRSS procedure used on the transfer functions.
- TF-Summation provides reasonable agreement with the overall shape of the target spectra but the amplitudes are somewhat lower.
- Response-Simulation provides the most accurate procedure as evidenced by the results.
- The accuracy of TF-SRSS and Response-Summation methods could further deteriorate when accounting for the effects of torsion and rocking in the structure, SSI, base slab flexibility and detailed FE model that were not considered in this study.

## REFERENCES

- Abrahamson, N. (2006). "Program on Technology Innovation: Spatial Coherency for Soil-Structure Interaction," *Electric Power Research Institute*, Palo Alto, USA, TR-1014101.
- EPRI (2007a). "Program on Technology Innovation: Effects of Spatial Incoherence on Seismic Ground Motions," *Electric Power Research Institute*, Palo Alto, USA, TR-1015110.
- EPRI (2007b). "Program on Technology Innovation: Validation of CLASSI and SASSI Codes to Treat Seismic Wave Incoherence in Soil-Structure Interaction (SSI) Analysis of Nuclear Power Plant Structures," *Electric Power Research Institute*, Palo Alto, USA, TR-1015111.
- Lysmer, J., Tabatabaie, M., Tajirian, F., Vahdani, S. and Ostadan, F. (1981). "SASSI – A System for Analysis of Soil-Structure Interaction," *Report No. UCB/GT/81-02, Geotechnical Engineering, Department of Civil Engineering, University of California, Berkeley, USA.*
- MTR/SASSI (2015). "System for Analysis of Soil-Structure Interaction," *Version 9.4.6.7, MTR & Associates, Inc., Lafayette, California.*
- Tabatabaie, M. (2014). "SASSI FE Program for Seismic Response Analysis of Nuclear Containment Structures," *Chapter 22: on Infrastructure Systems for Nuclear Energy*, John Wiley & Sons Ltd.
- Tseng, W. S. and Lilhanand, K. I. (1997). "Soil-Structure Interaction Analysis Incorporating Spatial Incoherence of Ground Motions," *Electric Power Research Institute*, Palo Alto, USA, TR-102631.
- Wong, H. L. and Luco, J. E. (1980). "Soil Structure Interaction: A Linear Continuum Mechanics Approach (CLASSI)," *Report CE, Department of Civil Engineering, University of Southern California, Los Angeles, USA.*



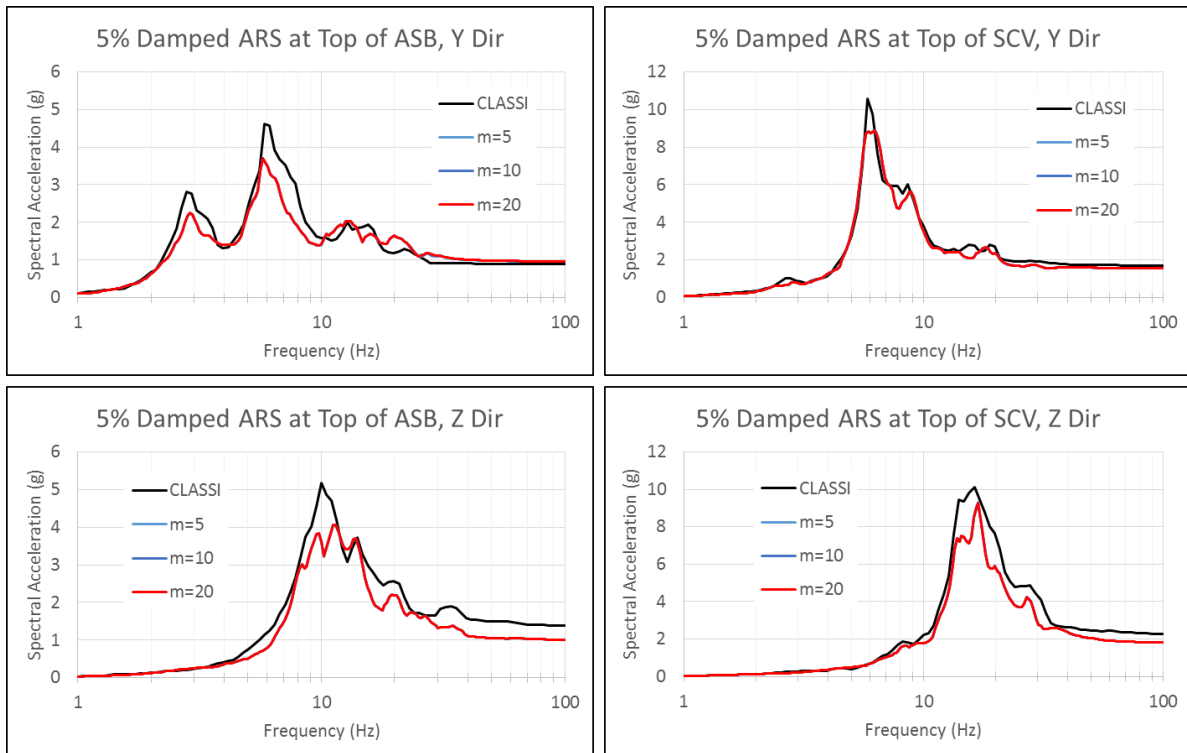


Figure 5. Incoherent Response Comparisons of 5, 10 and 20 Modes, Transfer Function SRSS

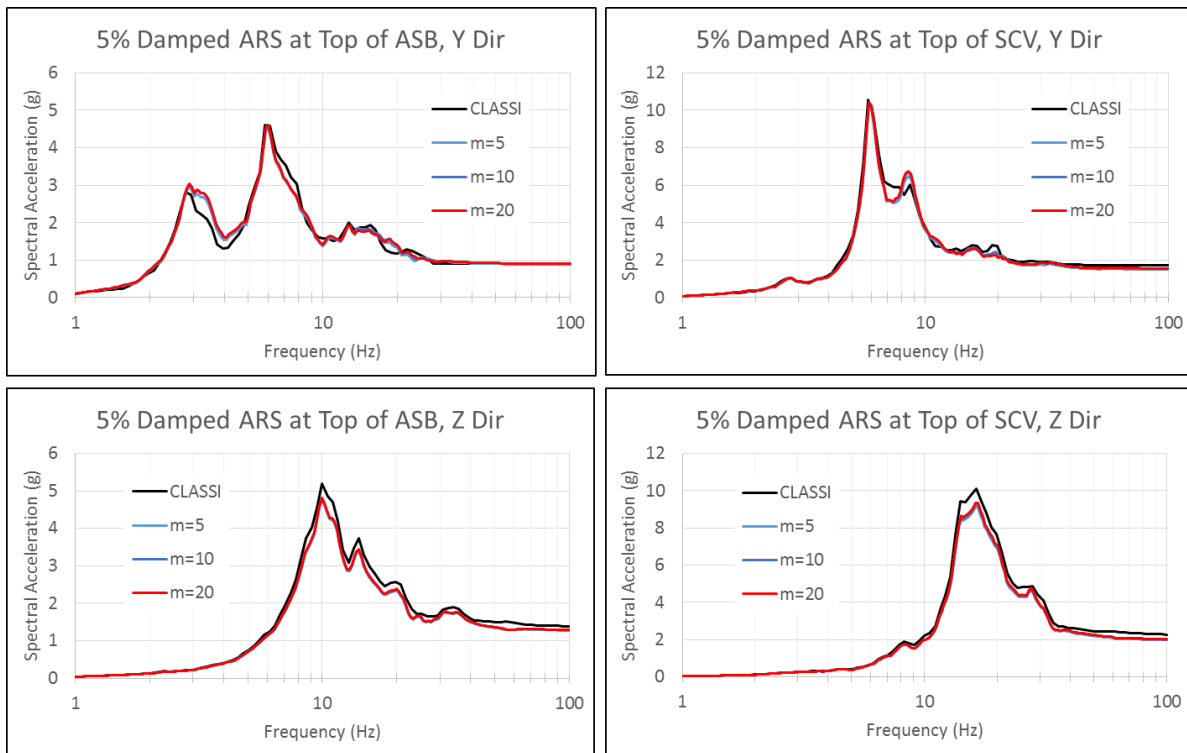


Figure 6. Incoherent Response Comparisons of 5, 10 and 20 Modes, TF-Summation

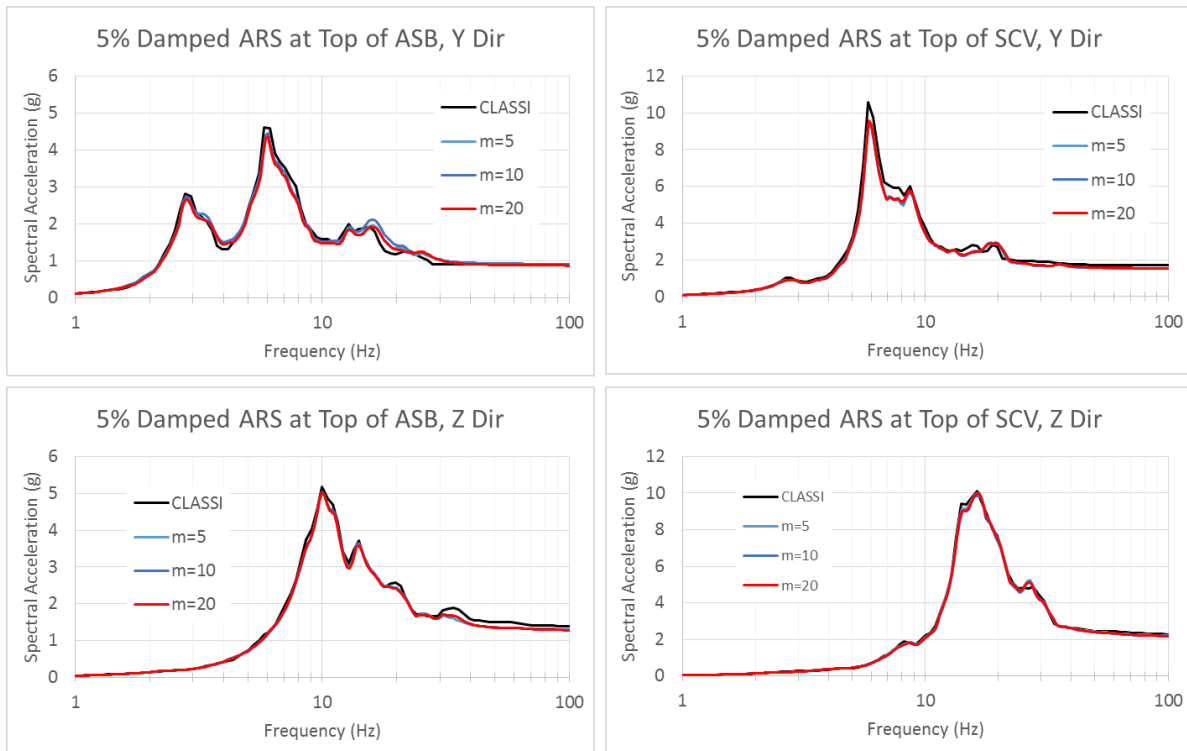


Figure 7. Incoherent Response Comparisons of 5, 10 and 20 Modes, Response-Simulation (Mean of 20 Simulations)

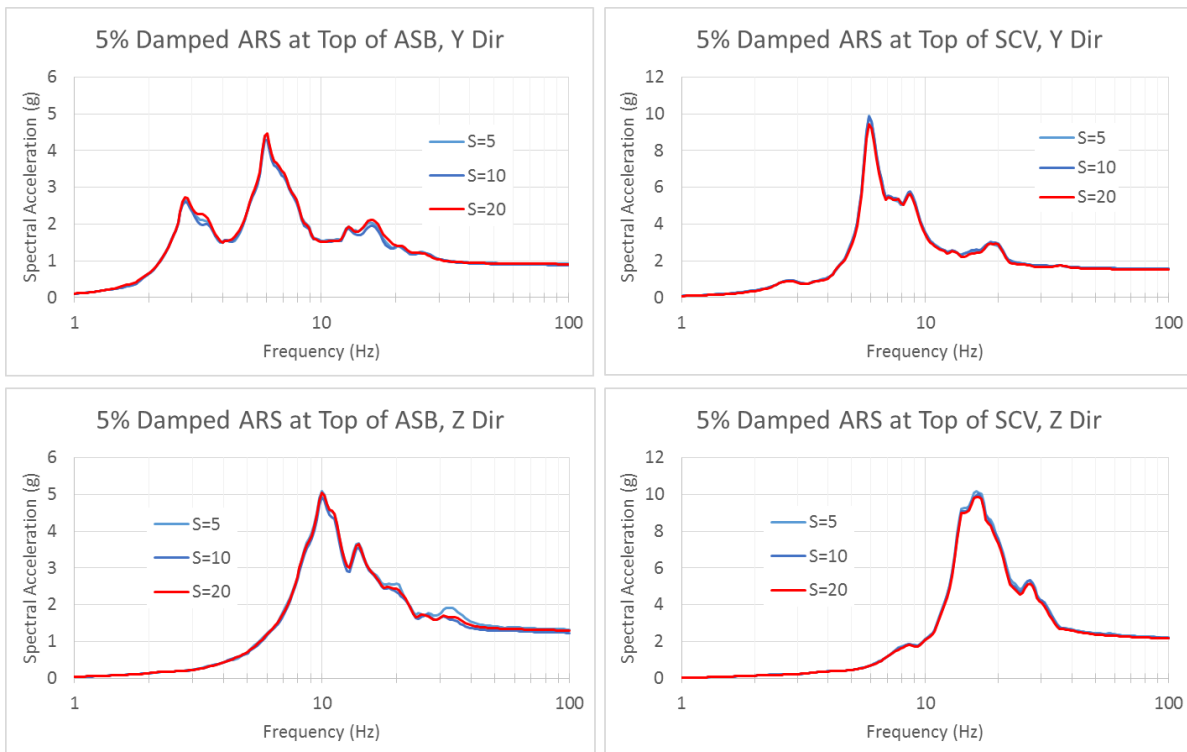


Figure 8. Incoherent Response Comparisons of 5, 10 and 20 Simulations, Response-Simulation (10 Modes considered)