VIBRATION ANALYSIS OF FOUNDATIONS ON LAYERED MEDIA

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ABSTRACT

A general procedure based on a new substructuring technique, the flexible volume method, is applied to the solution of foundation vibration problems. The procedure is capable of handling foundations with complex geometries, and multiple flexible foundations of arbitrary shape founded on the surface of, or embedded in, layered viscoelastic soils and subjected to harmonic or transient loadings. The methodology is implemented in a system of computer programs called SASSI. The procedures for computing the necessary three-dimensional impedance matrices and the overall response are summarized. To demonstrate the effectiveness and accuracy of this method, SASSI is used to evaluate the dynamic response (compliance functions) of a rigid circular disk founded on the surface of a soil layer resting on rigid base rock. The results compare favorably with more rigorous continuum solutions. These results are also compared with the response of circular footings on a damped halfspace to show the effects of a fixed base on foundation response. To demonstrate the applicability of the procedure to practical problems, results are presented of a three-dimensional aircraft impact analysis on an embedded tunnel.

INTRODUCTION

The design of foundations for vibrating machines and foundations subject to other dynamic forces requires an accurate prediction of the foundation response to these loads. A complete and rigorous analysis must account for the following: the three-dimensional nature of the problem, foundation flexibility, material and radiation damping of soil, variation of soil properties with depth, embedment effects, and interaction effects between multiple foundations through the soil.

Several substructuring methods for vibration analysis of foundations are presented in the literature. The basic approach in all these methods is to partition the complete soil-structure system into two parts - - the structure and the soil. The soil medium is analyzed first and the impedance properties (dynamic stiffnesses) at the foundation-soil interface are established. In the second step, these

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impedances are incorporated with the equations of motion of the structural system, and the overall response is computed using standard dynamic analysis procedures. For the case of foundations founded at the surface of a homogeneous halfspace, the substructuring procedures are simple and economical since the impedance functions can be readily obtained directly from rigorous continuum solutions. However, in most cases, the solution to the impedance problem is not known a priori and must be obtained by performing a separate analysis, e.g., by the finite element method. Furthermore, the solution of a complicated impedance problem makes the substructuring methods less attractive as compared to the complete finite element methods.

In this paper, a general procedure based on a new substructuring method, the flexible volume method [Tabatabaie (6)], is applied to the solution of foundation vibration problems. The procedure allows vibration analysis to be performed on foundations with complex geometries. It is possible to compute the response of multiple flexible foundations of arbitrary shapes founded on the surface of, or embedded in, layered viscoelastic soils and subjected to harmonic or transient loadings. The actual foundation rigidity can also be modeled.

The methodology differs from other substructuring techniques in the manner in which the stiffness and the mass matrices of the foundation are partitioned from those of the soil. The complete soil-foundation system is divided into two substructures, the soil and the foundation. The foundation is modeled by standard finite elements, and the interaction is assumed to occur over the embedded volume rather than at the boundary, i.e., at all foundation nodes below grade. The mass and stiffness of the foundation are reduced by the corresponding properties of the volume of soil excavated, but are retained within the halfspace. Thus, the impedance problem is reduced to a series of axisymmetric solutions of the response of a layered site to point loads [Tajirian (7)]. The above methodology is implemented in a system of interrelated computer program modules called SASSI (5).

The flexible volume method and the procedures for computing the necessary three-dimensional impedance matrices and the overall response are summarized below. To demonstrate the effectiveness and accuracy of this method, SASSI is used to evaluate the dynamic response (compliance functions) of a rigid circular disk founded on the surface of a soil layer resting on rigid-base rock. The results are compared with the more rigorous continuum solutions obtained using the computer program LUCON (4).

METHODOLOGY FOR GENERAL THREE-DIMENSIONAL FOUNDATION VIBRATION ANALYSIS

Formulation of the Flexible Volume Method

The detailed formulation of the flexible volume method for analysis of soil-structure systems is given in References (6) and (7). The formulation below applies to forced vibration analysis of foundations. The system is solved in the frequency domain using the complex response method. Material damping is accounted for by using complex material moduli. Transient loadings are decomposed by Fast Fourier Transform techniques.

Fig. 1 represents a complete plane-strain soil-structure system discretized by finite elements. The foundation of this system is truncated at some far distance from the structure, and the effect of the remaining halfspace is accounted for by introducing a set of forces $\underline{Q}_b$ which act on the external boundary of the model. (The selected model is chosen for explicitness only and the method is not limited to plane-strain models nor to discretized foundations.)

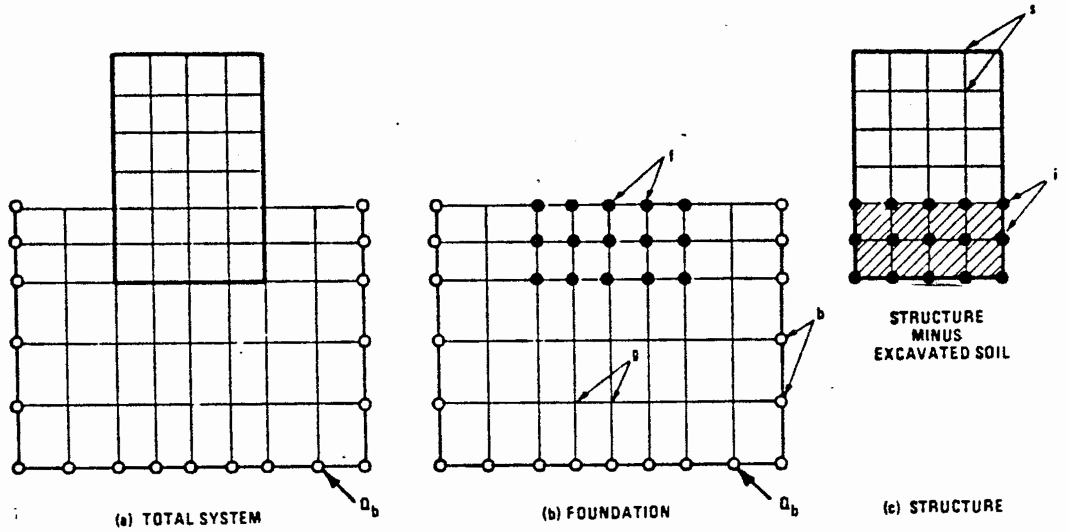

Figure 1. Substructuring of Interaction Model

The complete soil-structure system, Fig. 1a, is partitioned into two substructures, the foundation (Fig. 1b) and the structure (Fig. 1c). The mass and stiffness of the structure is reduced by the corresponding properties of the volume of soil excavated, but it is retained within the halfspace. Furthermore, the interaction is assumed to occur over a volume, i.e., at all basement nodes.

Using the complex response method, the discretized equation of motion for the complete system in Figure 1a can be formulated. This equation in the frequency domain can be written as:

$$\underline{C}^* \cdot \underline{U}^* = \underline{Q}_b^* \qquad (1)$$

where $\underline{U}^*$ and $\underline{Q}_b^*$ are the vectors of complex nodal point displacements and force amplitudes respectively and $\underline{C}^*$ is the complex frequency- dependent stiffness matrix, which can be written as

$$\underline{C}^* = \underline{K}^* - \omega^2 \underline{M} \qquad (2)$$

$\underline{M}$ and $\underline{K}^*$ are the total mass and complex stiffness matrices of the system, respectively, and are assembled using standard finite element techniques. In the above equations, the superscript "*" denotes a complex term. In the following derivation, this superscript is omitted with the understanding that all stiffness terms, displacements, and forces are complex, unless otherwise stated.

Similar equations of motion can be written for each of the substructures in Fig. 1. The following subscripts are introduced to refer to the degrees of freedom (DOF) associated with different nodes:

Subscript	Node
s	superstructure
i	basement
f	excavated soil
b	external boundary
g	remaining soil

The equation of motion for the soil (substructure 1) can be written as

$$\begin{bmatrix} \underline{C}_{ff} & \underline{C}_{fg} & \underline{C}_{fb} \\ \underline{C}_{gf} & \underline{C}_{gg} & \underline{C}_{gb} \\ \underline{C}_{bf} & \underline{C}_{bg} & \underline{C}_{bb} \end{bmatrix} \begin{Bmatrix} \underline{U}_f \\ \underline{U}_g \\ \underline{U}_b \end{Bmatrix} = \begin{Bmatrix} \underline{Q}_f \\ \underline{0} \\ \underline{Q}_b \end{Bmatrix} \tag{3}$$

where $\underline{Q}_f$ are the interaction forces from the structure. Similarly, the equation of motion for the structure (substructure 2) can be written

$$\begin{bmatrix} \underline{C}_{ss} & \underline{C}_{si} \\ \underline{C}_{is} & (\underline{C}_{ii}-\underline{C}_{ff}) \end{bmatrix} \begin{Bmatrix} \underline{U}_s \\ \underline{U}_i \end{Bmatrix} = \begin{Bmatrix} \underline{F}_s \\ \underline{F}_i+\underline{Q}_i \end{Bmatrix} \tag{4}$$

where $\underline{F}_s$ and $\underline{F}_i$ are the amplitudes of the external forces at the superstructure and basement nodes, respectively. Compatibility of displacements and equilibrium of forces at the soil-structure interface require the following conditions:

$$\underline{U}_i = \underline{U}_f \tag{5}$$

$$\underline{Q}_i + \underline{Q}_f = \underline{0} \tag{6}$$

By substituting Eq. (6) into (4), we obtain

$$\begin{bmatrix} \underline{C}_{ss} & \underline{C}_{si} \\ \underline{C}_{is} & (\underline{C}_{ii}-\underline{C}_{ff}) \end{bmatrix} \begin{Bmatrix} \underline{U}_s \\ \underline{U}_i \end{Bmatrix} = \begin{Bmatrix} \underline{F}_s \\ \underline{F}_i-\underline{Q}_f \end{Bmatrix} \tag{7}$$

The term $(\underline{C}_{ii}-\underline{C}_{ff})$ simply indicates the stated partitioning according to which the stiffness and mass of the excavated soil are subtracted from the stiffness of the structure.

If any existing rock boundary is at rest and the truncated external boundary is selected infinitely far from the loaded foundation, we can assume that

$$\underline{Q}_b = \underline{0} \tag{8}$$

By substituting Eq. (8) into (3) and partitioning, we obtain:

$$\begin{bmatrix} \underline{C}_{ff} & \underline{C}_{fg} & \underline{C}_{fb} \\ \underline{C}_{gf} & \underline{C}_{gg} & \underline{C}_{gb} \\ \underline{C}_{bf} & \underline{C}_{bg} & \underline{C}_{bb} \end{bmatrix} \begin{Bmatrix} \underline{U}_f \\ \underline{U}_g \\ \underline{U}_b \end{Bmatrix} = \begin{Bmatrix} \underline{Q}_f \\ \underline{0} \\ \underline{0} \end{Bmatrix} \tag{9}$$

According to the above partitioning $\underline{U}_g$ and $\underline{U}_b$ can be eliminated and the relationship between $\underline{U}_f$ and $\underline{Q}_f$ can be expressed in the form

$$\underline{Q}_f = \underline{X}_f \cdot \underline{U}_f \tag{10}$$

$\underline{X}_f$ is a frequency-dependent matrix which represents the dynamic stiffness of the foundation soil at the interaction nodes. $\underline{X}_f$ will be referred to as the impedance matrix. An effective method for determining this matrix without using the large matrix in Eq. 9 is described in the next section.

Substitutions of Eqs. (10), (5), and (6) into Eq. (7) results in

$$\begin{bmatrix} \underline{C}_{ss} & \underline{C}_{si} \\ \underline{C}_{is} & (\underline{C}_{ii}-\underline{C}_{ff}+\underline{X}_f) \end{bmatrix} \begin{Bmatrix} \underline{U}_s \\ \underline{U}_i \end{Bmatrix} = \begin{Bmatrix} \underline{F}_s \\ \underline{F}_i \end{Bmatrix} \tag{11}$$

According to this formulation, the solution of the foundation vibration problem reduces to two steps (for each frequency):

1. Solve the impedance problem, Eq. 10, to determine the matrix $\underline{X}_f$.

2. Solve the structural problem, Eq. 11. This involves forming the complex stiffness matrices and load vector and solving Eq. 11 for the final displacements using standard equation solvers.

Formulation of the Impedance Matrix

According to the definition of flexibility and stiffness matrices, the impedance (dynamic stiffness) matrix, $\underline{X}_f$, for the interaction degrees of freedom can be determined as the inverse of the dynamic flexibility matrix, $\underline{F}_f$, i.e.,

$$\underline{X}_f = \underline{F}_f^{-1} \tag{12}$$

$\underline{F}_f$ is a full symmetric complex matrix. An efficient in-place inversion subroutine (8) is currently used for such operation. This method is called the "direct method" for computing the impedance matrix. Other efficient methods for computing the impedance matrix have been developed (6) and (8). These include procedures for computing the impedance matrices of symmetric and antisymmetric systems.

Formulation of the Flexibility Matrix

Procedures for determining the flexibility matrix for three-dimensional systems are described by Tajirian (7). The basic problem in determining the dynamic flexibility matrix is to find the response of a layered halfspace to a harmonic point load. Each column of the flexibility matrix is formed by applying a unit point load at the interaction degree-of-feeedom associated with that column and by computing the resulting displacements at all the interaction nodes.

For layered sites, these displacements can be obtained from the axisymmetric model shown in Fig. 2. This model consists of a central core of special cylindrical axisymmetric finite elements connected at the perimeter to a semi-infinite layered zone which is represented by axisymmetric transmitting boundaries (3) and (10). Either the lower boundary can be fixed or a halfspace can be simulated by using the variable depth and viscous boundary methods (1), (6) and (7).

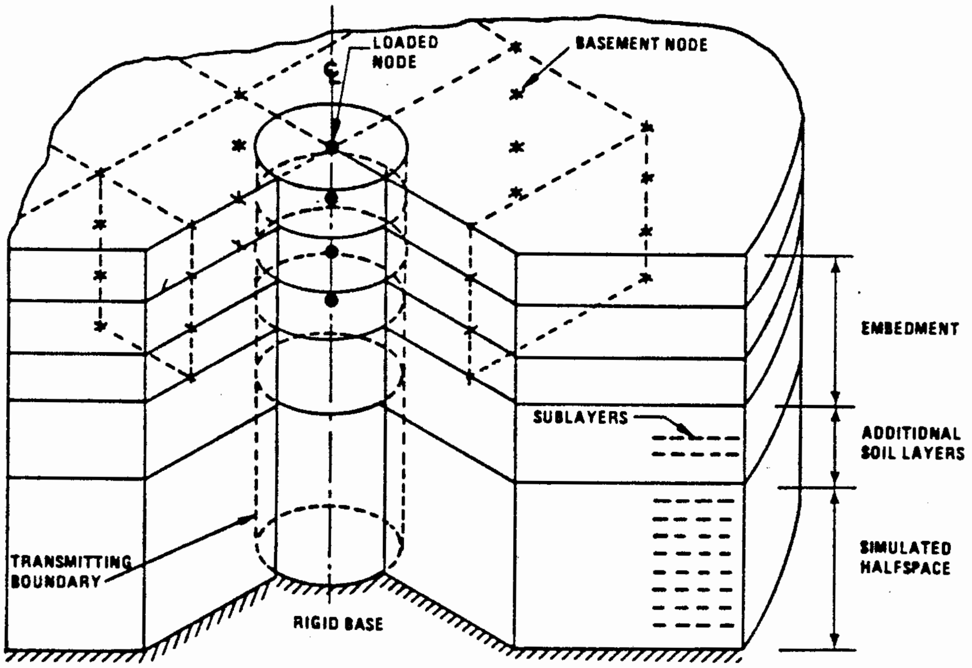

Figure 2. Axisymmetric Model for Impedance Analysis

From this model displacement amplitudes can be obtained both at the central nodes and at any point outside the cylindrical elements, i.e., at all interaction nodes. These displacements which are computed in a cylindrical coordinate system are transformed to the global cartesian coordinate system using standard transformation procedures.

CASE STUDIES

Case Study 1: Compliance Functions For Circular Disk on a Layer Overlying a Rigid Base

In this section, the accuracy of the above methodology is tested by comparing SASSI results with known solutions for the problem of forced vibration of rigid foundations on viscoelastic layered media.

Compliance functions for a rigid circular disk on a uniform layer of finite thickness overlying a horizontal rigid base were obtained using the computer program LUCON (4). This program uses a special formulation of the field problem in terms of Green's functions and can handle a layered viscoelastic halfspace.

The same problem was solved using SASSI. Fig. 3 shows the geometry, the material properties, and the finite element mesh used in the SASSI analysis. Because the results of the analysis are presented in terms of the dimensionless frequency ratio $A_o = \omega r/V_s$, the parameters of the problem can be selected arbitrarily as long as the frequencies of analysis, ω, are adjusted accordingly to cover the presented range of A_o.

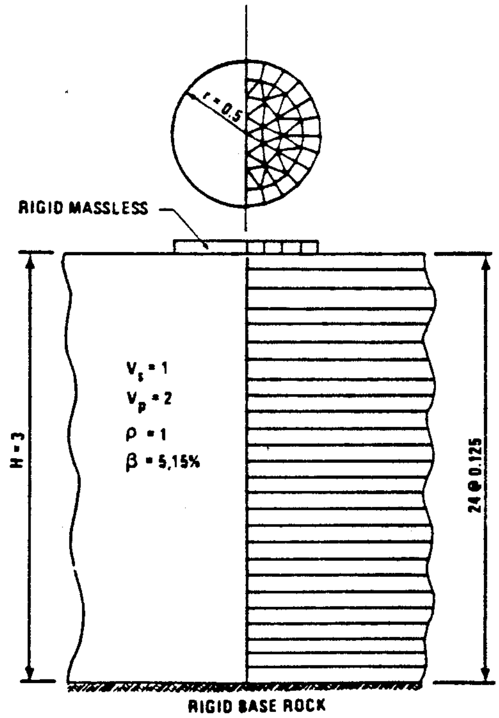

Figure 3. SASSI Model for Computing Compliance Functions

The compliance functions for 5 and 15 percent layer damping obtained using the SASSI procedure are compared with comparable LUCON results. These results are also presented along with damped halfspace solutions (9) to investigate the effectiveness of the model consisting of a finite layer on a rigid base rock (Fig. 3) in simulating halfspace conditions. Figs. 4 and 5 show the vertical compliance functions for 5% and 15% damping, respectively. Excellent agreement exists between SASSI and LUCON results except for the real part of the 5% compliance at A_o values of less than 0.2. Considerable deviation of the results from halfspace solution can be seen especially near the peaks corresponding to vertical natural frequencies of the system. These frequencies can be obtained from

$$F_{vnf} = (2n-1)\, V_p/(4H) \quad , n=1,2,3...$$

For the present problem (V_p = 2, H = 3), F_{vnf} = 0.166, 0.500, 0.833, ...
and thus: A_{vnf} = 0.524, 1.571, 2.618, ...

However, the results seem to be less sensitive to occurrence of the peaks at higher modes especially when the damping is high. In general, the vertical compliance of the model shown in Fig. 3 poorly represents the damped halfspace results at A_o less than 1.5.

Comparisons of horizontal compliance functions are shown in Figs. 6 and 7. Both methods yield essentially the same results. The rigid-base results tend to better match the halfspace results for the horizontal case than for the vertical case. This is in part due to the fact that the peaks corresponding to the horizontal natural frequencies of the layer on rigid base start from much lower frequencies.

$$F_{hnf} = (2n-1)\, V_s/(4H) \quad n=1,2,3. \;...$$

thus for V_s = 1, H = 3, we obtain F_{hnf} = 0.083, 0.250, 0.417, ...
and A_{hnf} = 0.262, 0.785, 1.309, ...

Comparison of the results for rocking and torsional cases for 5 and 15 percent damping, as shown in Figs. 8 through 11, indicate very good agreement between SASSI and LUCON results. Furthermore, it appears that the rocking and torsional compliance functions obtained from the model shown in Fig. 3 can be used to represent those of the damped halfspace case with good accuracy. This is in agreement with the findings of other investigators who have shown that the compliance functions for rotational sources are not influenced by the presence of a rigid base when the depth to the base is greater than three times the foundation radius (3). This is due to the destructive interference of waves emanating from the footing-soil interface which limit the effective depth of penetration of the generated waves.

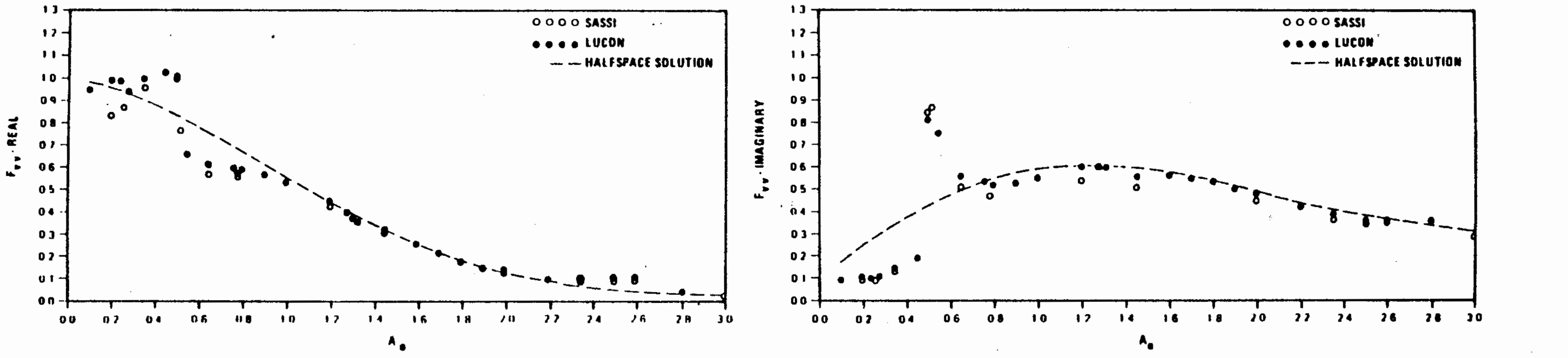

Figure 4. Vertical Compliances, Damping = 5%

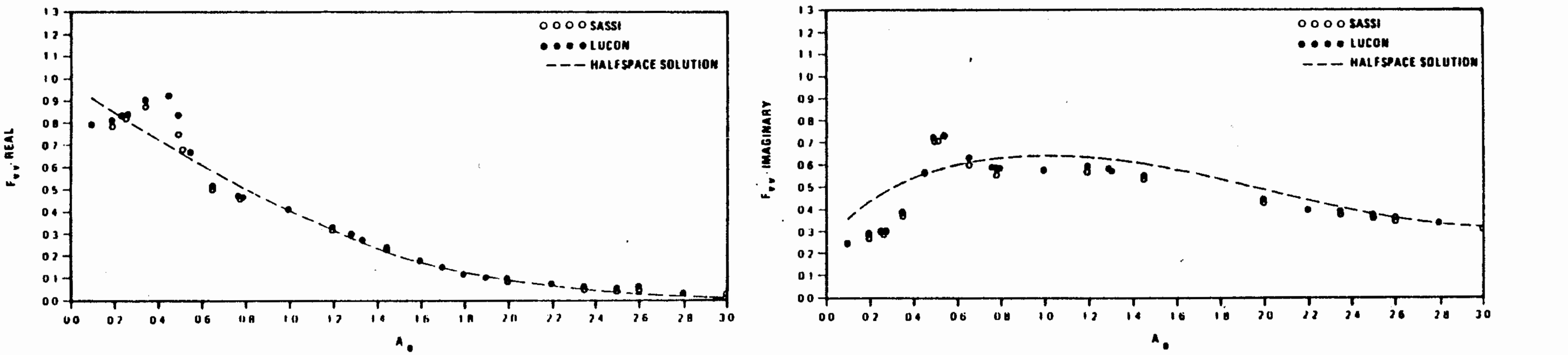

Figure 5. Vertical Compliances, Damping = 15%

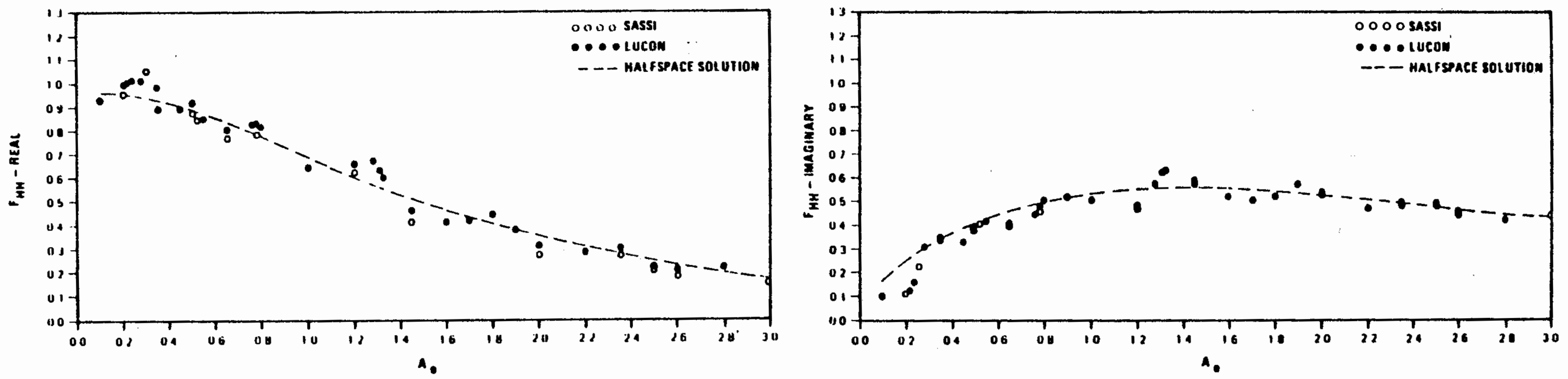

Figure 6. Horizontal Compliances, Damping = 5%

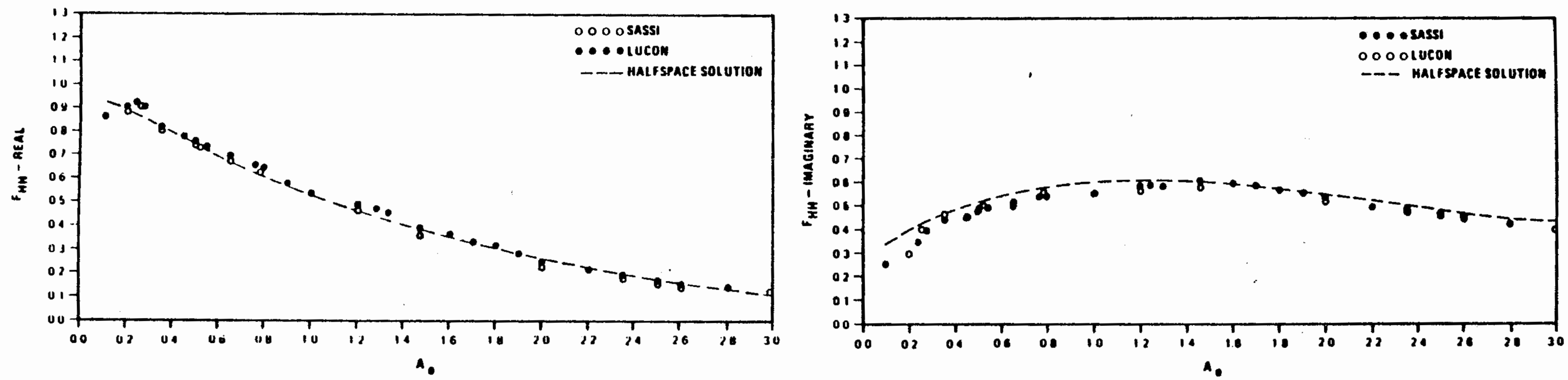

Figure 7. Horizontal Compliances, Damping = 15%

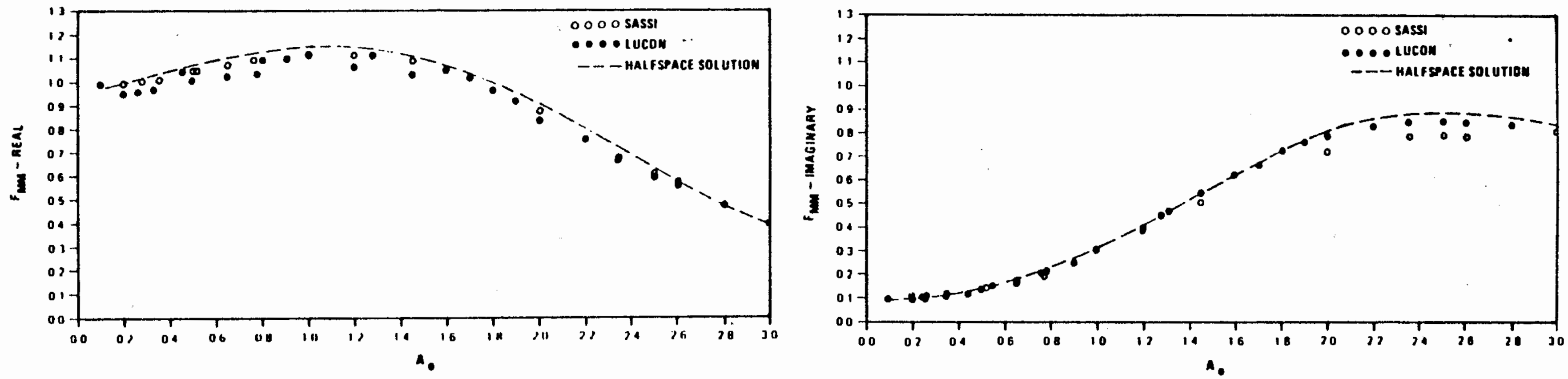

Figure 8. Rocking Compliances, Damping = 5%

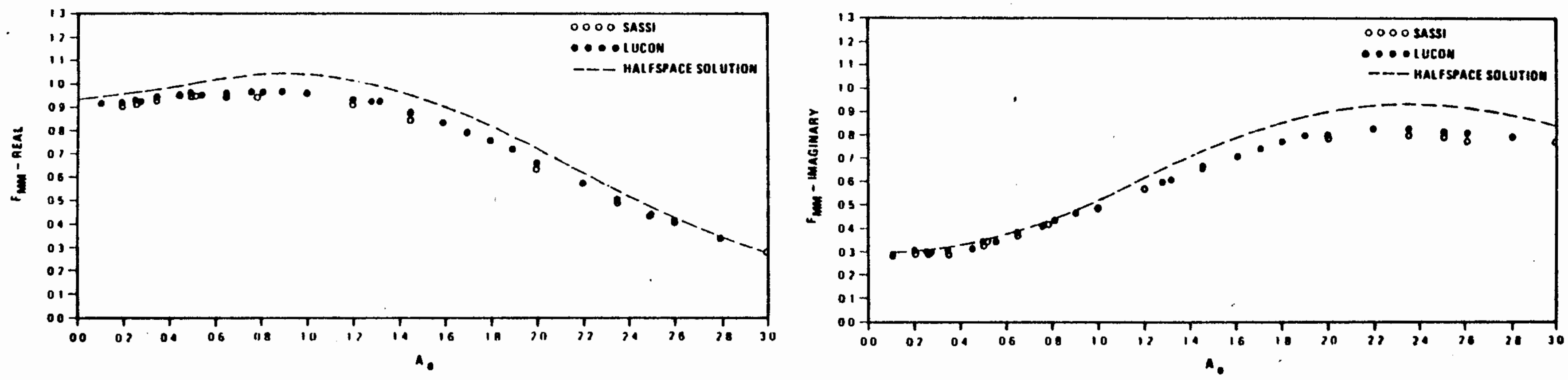

Figure 9. Rocking Compliances, Damping = 15%

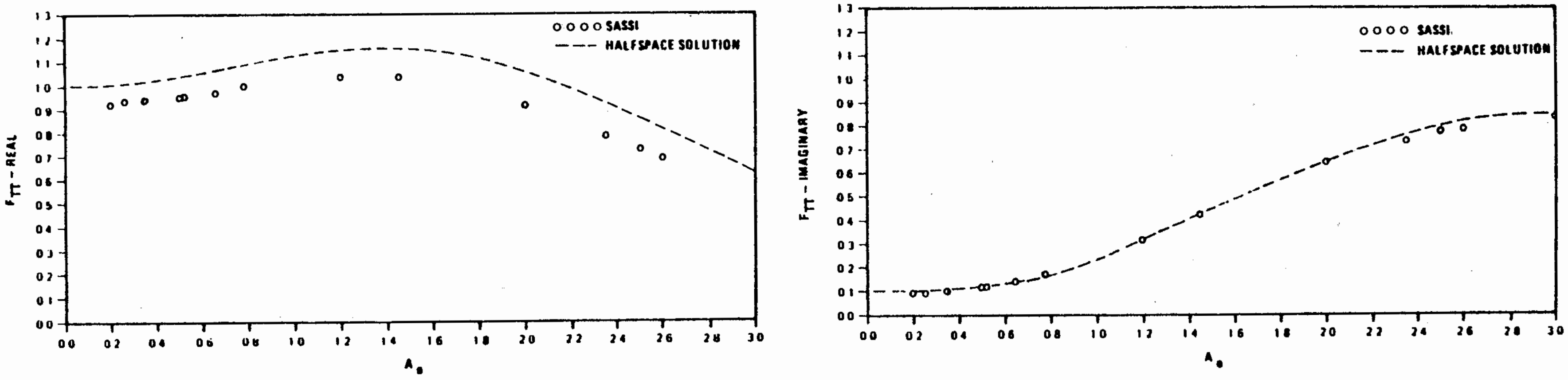

Figure 10. Torsional Compliances, Damping = 5%

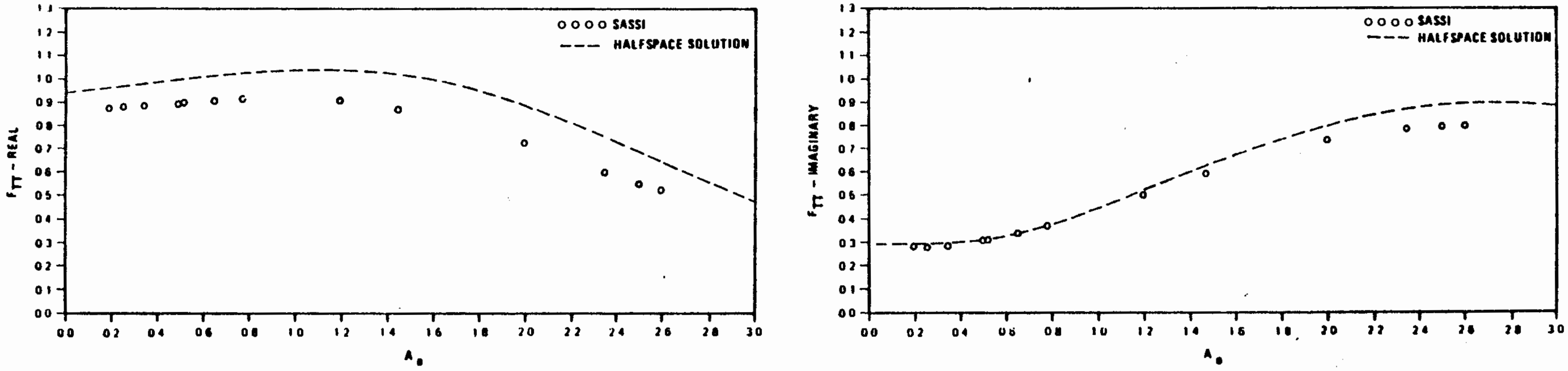

Figure 11. Torsional Compliances, Damping = 15%

Case Study 2: Three-Dimensional Analysis of Airplane Impact on an Underground Cable Tunnel and Protective Slab

The second case study is a three-dimensional analysis of an underground cable tunnel. The response of the tunnel to an aircraft impact on a protective slab at grade is computed.

The tunnel which runs between a reactor and control building at a nuclear power plant is shown in Figs. 12 and 13. The tunnel has a height of about 15 ft and is 35.8 ft wide. It is designed with a protective slab about 3 ft thick at the ground level and with an intermediate layer of earth about 3.25 ft thick. To prevent impact on the sides of the tunnel, the protective slab is wider than the tunnel. The thickness of the protective slab is designed to prevent perforation in the direct loading area where an airplane might impact. Thus, the protective slab and the tunnel have to be designed to withstand stresses during the time-dependent loading shown in Fig. 14.

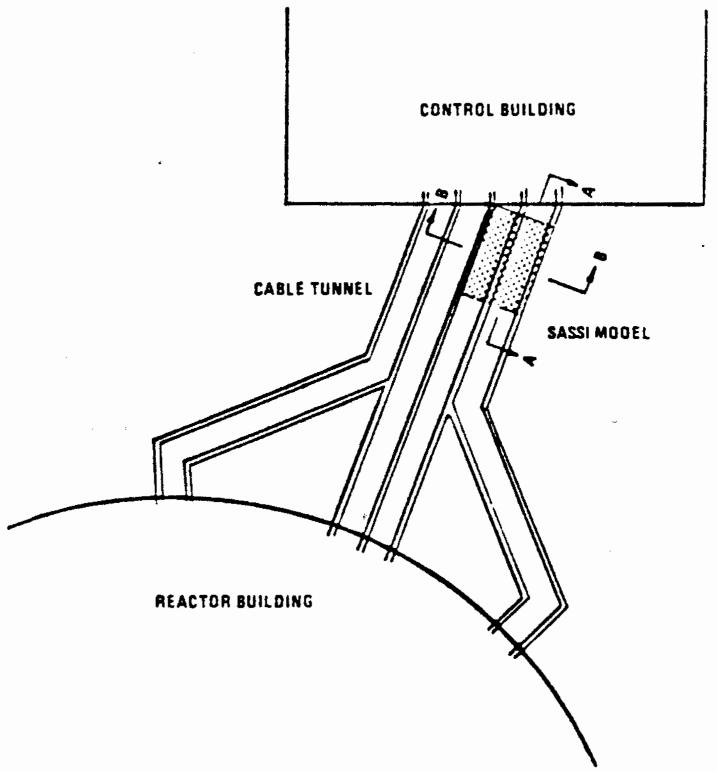

Figure 12. Plan View of Tunnel

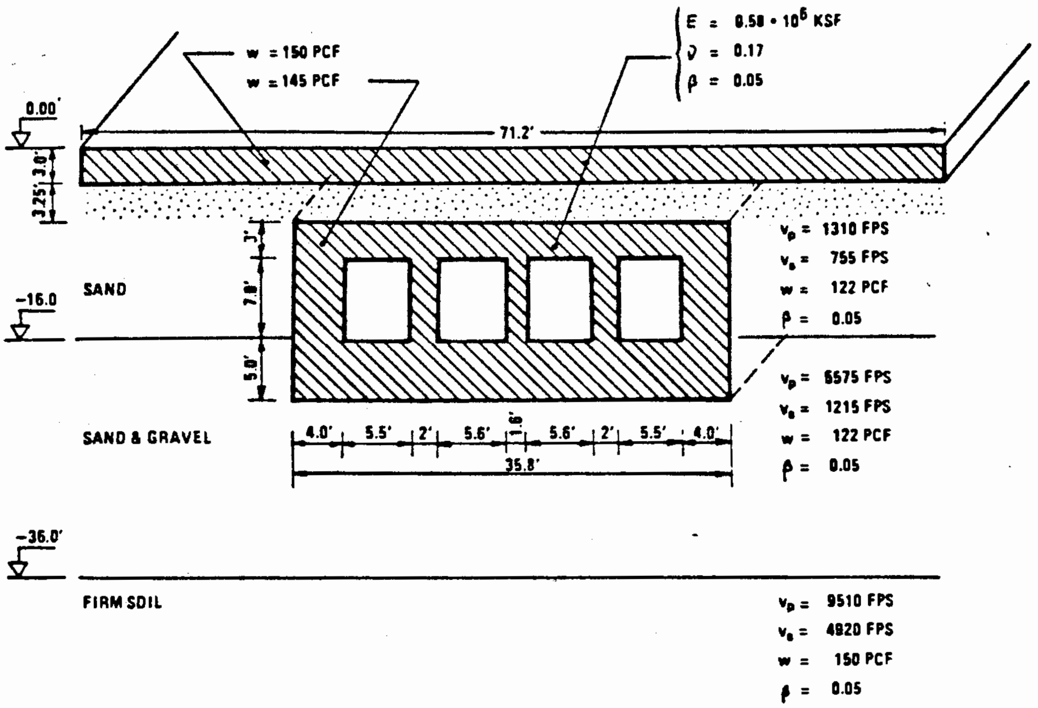

Figure 13. Section Through Tunnel

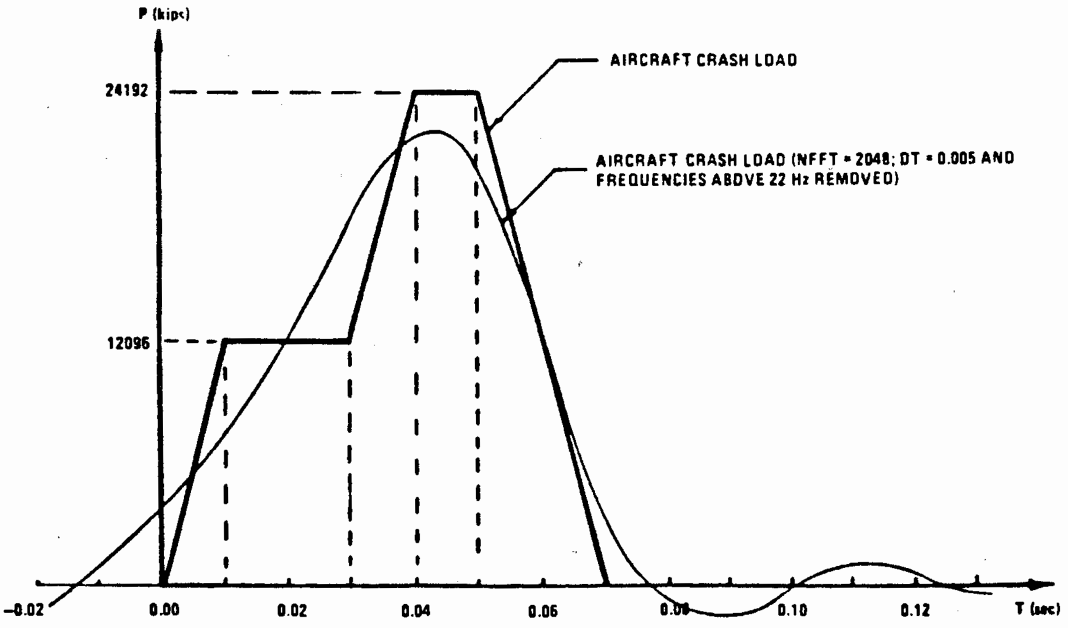

Figure 14. Input Time History

The soils around the tunnel consist of about 16 ft of sand underlain successively by 20 ft of sand and gravel and a deep bed of firm soils (Fig. 13).

A standard German aircraft crash loading was used as input. The loading was assumed to act vertically at the center of the protective slab and was distributed over an area of 75 ft^2. The maximum amplitude of the load was 24,192 kips. In the analysis, one quarter of the total load was distributed on the symmetric model of the prototype slab (Fig. 15).

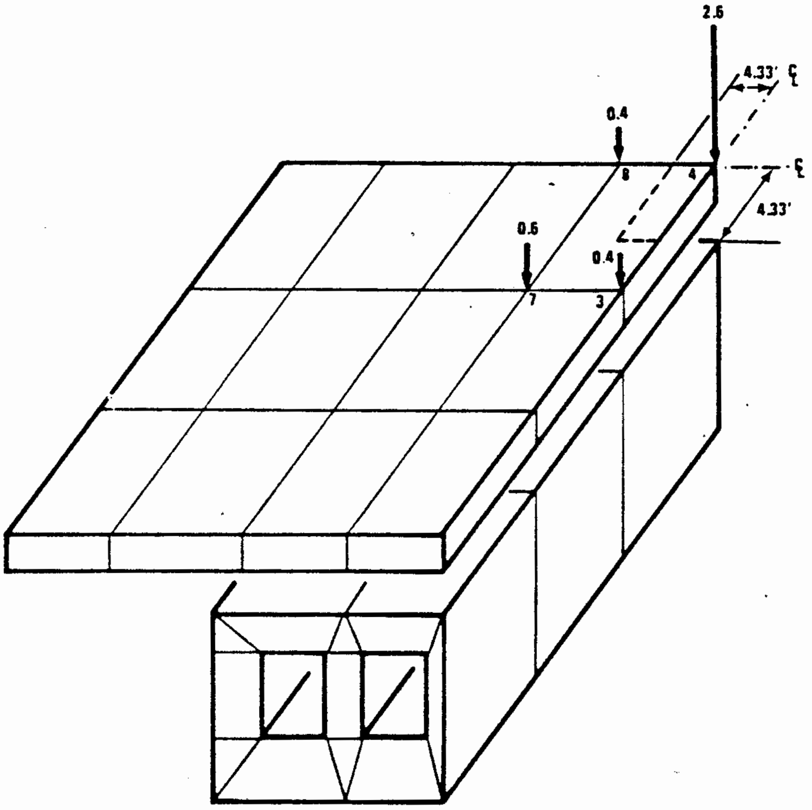

Figure 15. Input Load Distribution

The first 0.16 seconds of the load digitized at 0.005 seconds was used. The record consisted of 2048 points. The maximum frequency considered was 22 Hz. The filtered time history is compared with the original loading (Fig. 14).

The soil profile used in the SASSI analysis is shown in Fig. 16. Layer thicknesses were chosen to be less than one fifth of the shear wave length at the cutoff frequency of 22 Hz; i.e., allowable thickness $V_s/(5*22)$. The underlying halfspace (firmsoil) was simulated using 20 sublayers whose thicknesses vary with frequency attached to viscous dashpots.

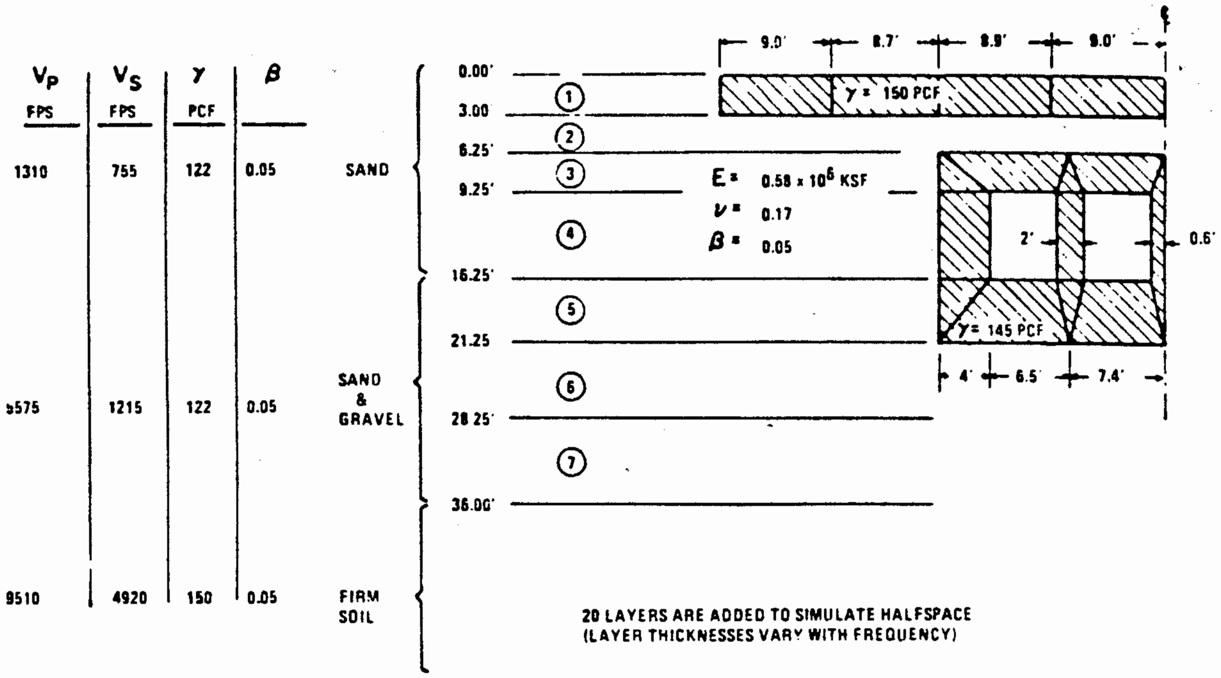

Figure 16. SASSI Model

The three-dimensional structural finite element model used in the SASSI analysis is shown in Fig. 17. Since advantage could be taken of symmetry about two vertical planes, only one quarter of the tunnel section was analyzed. Furthermore, it was assumed that the effects of impact on the tunnel and protective slab were minor at a distance of about 24 ft from the center of the loaded area. The protective slab and tunnel walls were modeled by special solid-brick elements which behave well in bending. The total model consists of 104 nodes, 45 structural elements, and 51 excavated soil elements.

Maximum vertical displacements were computed in the tunnel and the protective slab. The largest displacement was 0.63 in., at the center of the protective slab. The displacements in the tunnel below the loaded area did not exceed 0.07 in. Furthermore, at points away from the center of the loaded area the effect of impact was significantly reduced.

Vertical time histories at the center of the protective slab (node 4) and at the bottom of the tunnel (node 96) were compared. Such comparisons for displacements are shown in Fig. 18, and for accelerations in Fig. 19. As may be seen from these results the protective slab significantly reduced the motions in the tunnel.

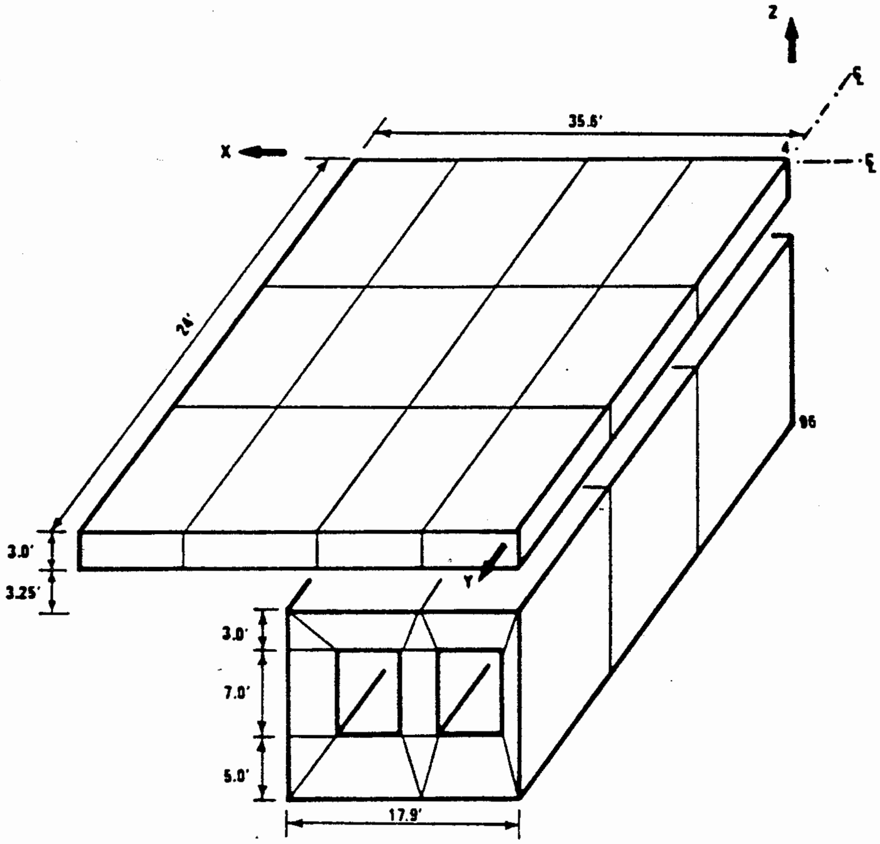

Figure 17. Three-Dimensional Finite Element Model

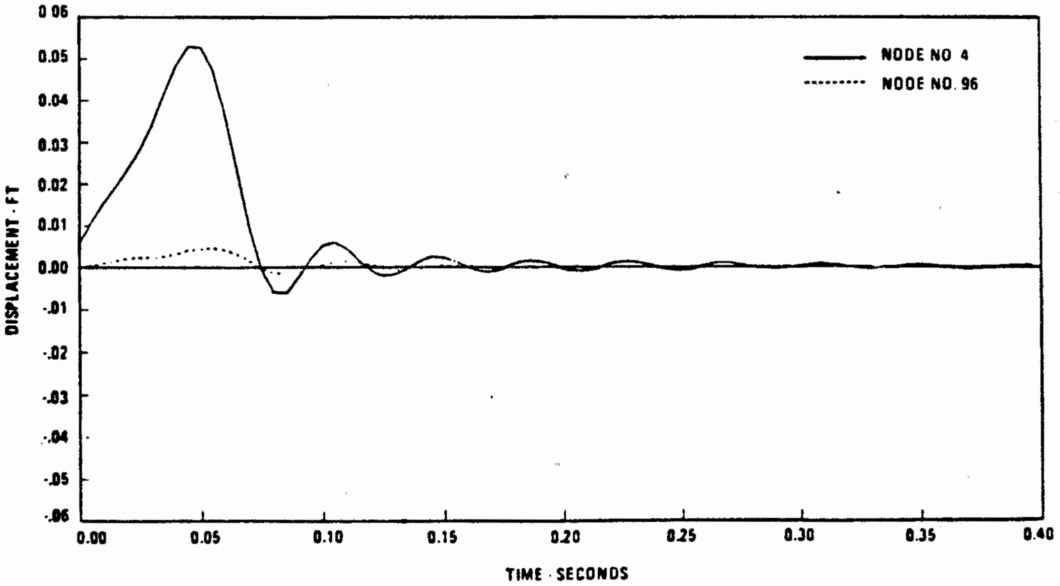

Figure 18. Comparison of Vertical Displacement Time History

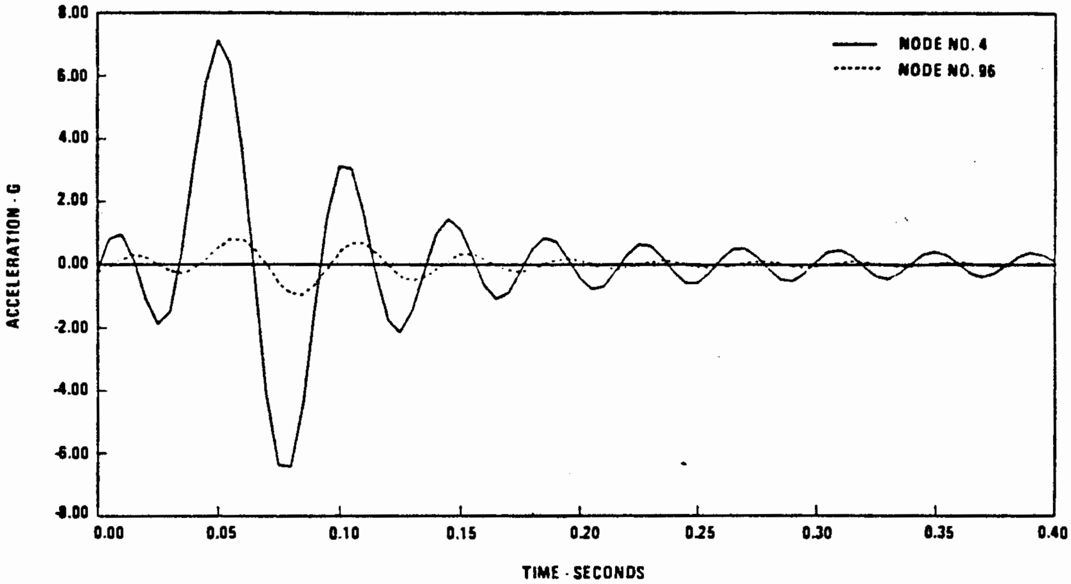

Figure 19. Comparison of Vertical Acceleration Time History

Vertical acceleration response spectra are shown in Fig. 20 (Node 4) and Fig. 21 (Node 96). The results indicate a peak in the response around the cutoff frequency (22 Hz). Thus, it may be necessary to repeat the entire analysis using a finer model which satisfied a higher frequency cutoff.

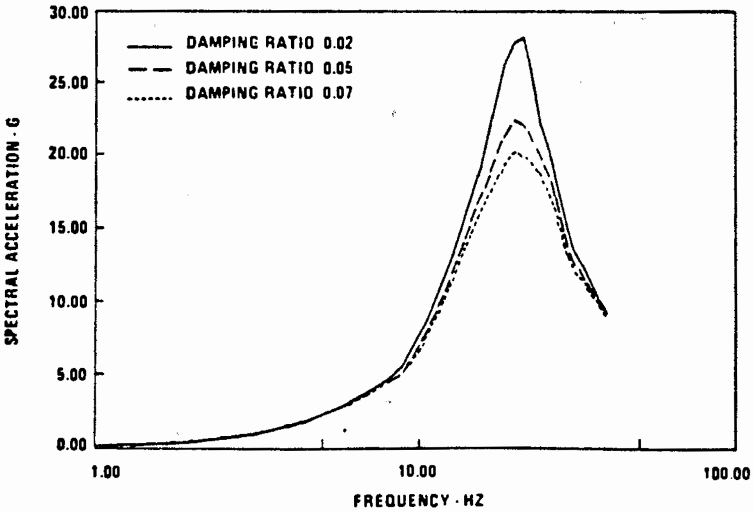

Figure 20. Absolute Vertical Acceleration Response Spectrum at Node 4

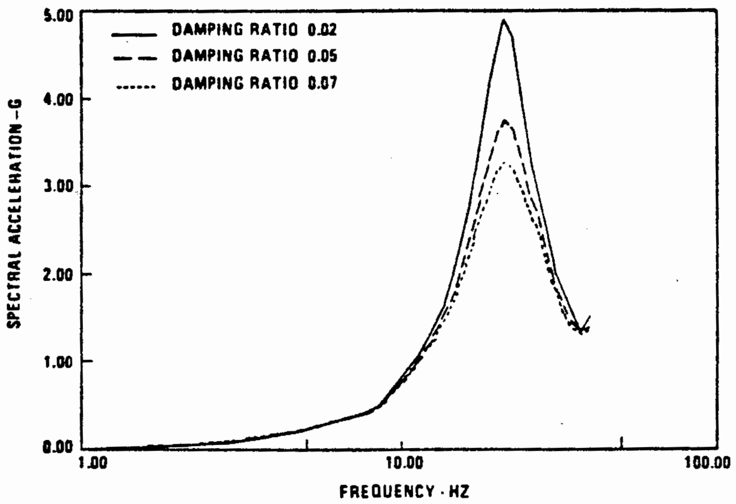

Figure 21. Absolute Vertical Acceleration Response Spectrum at Node 96

Comparisons of maximum velocities and accelerations at the same nodes are shown below. Again, the protective slab significantly reduced the motions in the tunnel.

	Node 4	Node 96
Maximum vertical displacement (ft)	0.053	0.004
Maximum vertical velocity (ft/sec)	2.80	0.33
Maximum vertical acceleration (g)	7.2	0.95

SUMMARY AND CONCLUSIONS

The flexible volume method was applied to the solution of general three-dimensional foundation vibration problems. The formulation can handle multiple flexible foundations with arbitrary shapes founded on the surface of, or embedded in, layered viscoelastic soils. Responses computed using this method compared favorably with more rigorous continuum solutions, demonstrating its accuracy. To illustrate the applicability of the procedure to more practical problems, the results of a three-dimensional analysis of an aircraft impact on a buried tunnel were presented.

ACKNOWLEDGMENTS

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APPENDIX. - REFERENCES

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