Time-Domain SSI Analysis of Typical Reactor Building using Frequency-Dependent Foundation Impedance Derived from SASSI

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ABSTRACT

In this paper we present a methodology for performing SSI analysis in time domain using distributed parameter foundation impedance (DPFI) model derived from SASSI analysis. First the foundation displacements and reaction forces in the x, y and z directions at each foundation interaction degree-of-freedom (DOF) are calculated from analysis of the total SSI system in frequency domain using SASSI. The results are then used to form matrix-valued, distributed foundation impedance (DFI) functions. In the second step, the resulting uncoupled, frequency-dependent DFI functions are linearized based on the response of a simple damped oscillator to develop the DPFI model. In the final step the DPFI model and scattered motions are incorporated in time-domain model of the structure using kinematic formulation. The accuracy of this procedure for the case of very stiff/rigid embedded foundation is demonstrated by analyzing the response of a typical pressurized water reactor (PWR) containment building subject to horizontal input motions. The comparison of results from one-step SASSI analysis of the total SSI system in frequency domain with those of ADINA analysis of the structure with DPFI model in time domain using direct integration method are presented and shown to be in good agreement.

INTRODUCTION

Three-dimensional seismic soil-structure interaction (SSI) analysis of nuclear reactor buildings is often performed in frequency domain using programs such as SASSI [1], etc. From foundation point of view, this enables the analyst to properly a) address the effects of wave propagation in unbounded soil media, b) incorporate strain-compatible soil modulus and damping properties and c) specify input motion in the free field using de-convolution method. However, in terms of structural modeling, frequency-domain programs are usually limited in their capabilities. For complex structures with many thousands of DOF's, it is desirable to model the structure using time domain programs such as ADINA [2], etc. with robust element libraries and capabilities that may be necessary to analyze aspects such as: non-linearity in the structure, coupled equipment and piping systems, post processing of force summations to calculate shear and overturning moments, etc.

The desire to break a complex problem into smaller more manageable systems has led to development of sub-structuring technique for dynamic response analyses of large SSI systems [3], [4], [5]. In sub-structuring method, the total SSI system is partitioned into two subsystems, namely the structure and foundation. The foundation is analyzed first, generally in frequency domain, and the foundation dynamic impedance and scattering properties are established at the structure/foundation interface. These properties are then used as boundary condition in dynamic analysis of the structure, which can be performed in time domain. This is ideal except that because two different dynamic solution methods (frequency versus time domain) are mixed to analyze the SSI system, significant difficulties often arises when complex-valued, frequency-dependent, impedance properties derived from analysis of the foundation in frequency domain are to be incorporated as boundary condition in time domain analyses of the structure (so called "handshake").

For structures supported on regular-shaped rigid mats, the foundation impedance properties are greatly simplified, i.e. the impedance matrix is reduced to a 6x6 diagonal matrix (off-diagonal terms are small and are ignored). This class of problems has been extensively studied by other researchers [6], [7], [8], [9], [10] among others and methods using constant spring-mass-dashpot (also referred to as lumped parameter foundation impedance) models have been devised to address the frequency/time-domain handshake. To account for the effects of mat flexibility, the authors proposed a new procedure for distributing the foundation impedance to all interaction DOF's. [11]. In the present paper, the same procedure (DPFI model) has been extended to embedded foundations and validated for a typical nuclear reactor building.

METHODOLOGY

The general dynamic equation of motion for a structure embedded in a semi-infinite halfspace and subject to a seismic input, as shown in Fig. 1(a) may be written in frequency domain as follows:

$$(-\omega^2\underline{M} + i\omega\underline{C} + \underline{K}) \,.\, \underline{U} = \underline{Q} \qquad (1)$$

Where $\underline{M}$, $\underline{C}$ and $\underline{K}$ are the total mass, damping and stiffness matrices, respectively, $\underline{Q}$ is the vector of external forces, $\underline{U}$ is the total displacement response vector, ω is circular frequency and $i=\sqrt{-1}$. For brevity in all later discussions all underscores for matrix or vector representation are dropped and the complex-valued, coefficient matrix in Eq. (1) is replaced by $K = -\omega^2\underline{M} + i\omega\underline{C} + \underline{K}$.

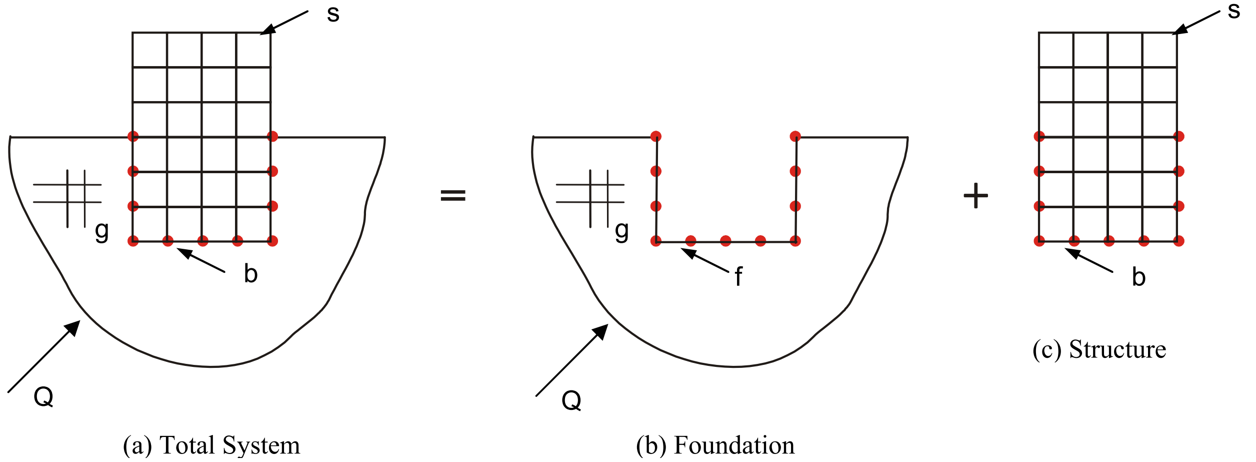


Fig. 1 – Illustration of sub-structuring method

By partitioning the total SSI system into two subsystems (foundation and structure, as shown in Fig. 1(b) and 1(c), respectively) and applying the sub-structuring formulation, Eq. (1) may be written as follows:

$$\begin{pmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb}+X_f \end{pmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ X_f \,.\, U_f \end{Bmatrix} \qquad (2)$$

Where "s", "b" and "f" denote the superstructure, basement and interaction DOF's, respectively. X_f is subgrade dynamic impedance matrix and U_f is vector of scattered displacements at interaction DOF's.

To solve Eq. (2) for the structure, the subgrade dynamic impedance (X_f) and scattered motions (U_f) at all foundation interaction DOF's are needed. Assuming that the foundation reaction forces, Q_b and interaction displacements, U_b are known from solution of total SSI system, Eq. (2) may be rearranged, as follows:

$$X_f \,.\, (U_f - U_b) = Q_b = [q^j/(u_f - u_b)^j] \,.\, (U_f - U_b) = D_f \,.\, (U_f - U_b) \qquad (3)$$

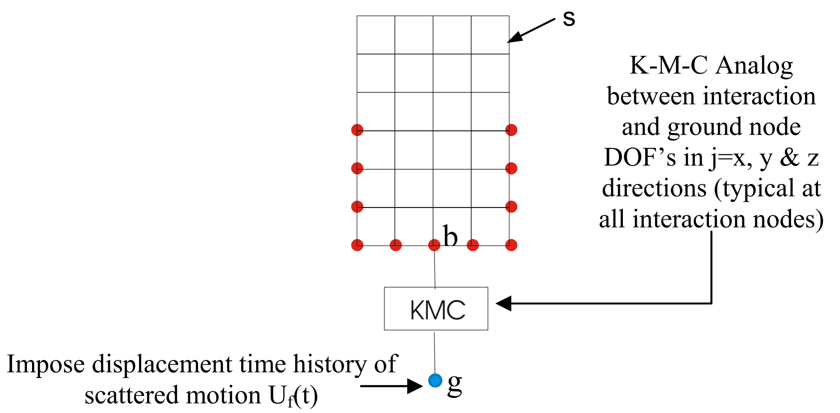
Where $D_f =[q^j/(u_f - u_b)^j]$ is a diagonal matrix representing the complex-valued, dynamic stiffness of foundation interaction DOF's; and q^j and u^j are the soil reaction force and displacement at interaction DOF j. Substituting D_f for X_f from Eq. (3) into Eq. (2), we obtain:

$$\begin{pmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb}+D_f \end{pmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ D_f \,.\, U_f \end{Bmatrix} \qquad (4)$$

As can be seen above, Eq. (4) is the same as Eq. (2) except that the full subgrade impedance matrix has now been replaced by a diagonal foundation impedance matrix consisting of uncoupled dynamic springs. Equation (4) offers an advantage over Eq. (2) in that D_f does not contain any coupling terms, i.e. it can be used to develop equivalent foundation dynamic springs for time domain analysis. It should be noted that Eq. (4) will provide exact solution to the SSI problem, as shown in Fig. 1(a), only if the soil reaction forces (Q_b) and interaction displacements (U_b) are known. Because Q_b and U_b depend on the foundation configuration and dynamic loading, D_f is referred to as "distributed foundation impedance (DFI)".

DFI is calculated by first solving the total SSI system in frequency domain to obtain Q_b and U_b at each interaction DOF j and then substituting them together with the scattered motions in Eq. (3) to calculate D_f. Alternatively, for structures having very stiff/rigid basement, DFI can be calculated with reasonable accuracy from impedance analysis of rigid massless foundation (the same model is also used to calculate scattered motions). In this case, unit-amplitude harmonic horizontal and vertical forces are applied separately at the center of base slab and the resulting reaction forces and displacements at all interaction DOF's are calculated and used to construct DFI matrix. DFI is a diagonal matrix in which each element constitutes a complex-valued, frequency-dependent, discrete function representing dynamic impedance at an interaction DOF. The real part of the dynamic impedance is associated with foundation stiffness and the imaginary part represents foundation damping. Because frequency-dependent foundation stiffness and damping parameters can not be directly used in time domain analyses, they are linearized based on the response of a single damped oscillator system having constant spring (k^j), dashpot (c^j) and mass (m^j) properties. The resulting system is referred to as DPFI or distributed KMC model.

DPFI model may be implemented in time domain analysis of the structure using either kinematic or inertial formulation [11]. In kinematic formulation, which is used herein, the DPFI model consists of a series of KMC analogs placed between the interaction and fictitious ground node DOF's in the x, y and z directions. The ground nodes are then driven by displacement time histories of scattered motions, as shown in Fig. 2 below. It should be noted that in this implementation, the virtual foundation mass, m^j, should be included in the coefficient matrix with negative off-diagonal terms similar to foundation stiffness and not simply as lumped masses.



$$\begin{cases} k_{f,j} = \begin{pmatrix} k^j & -k^j \\ -k^j & k^j \end{pmatrix} \\ m_{f,j} = \begin{pmatrix} m^j & -m^j \\ -m^j & m^j \end{pmatrix} \\ c_{f,j} = \begin{pmatrix} c^j & -c^j \\ -c^j & c^j \end{pmatrix} \end{cases}$$

Fig. 2 - Implementation of DPFI model in time domain model of structure using kinematic formulation

VALIDATION

The accuracy of this methodology is demonstrated by analyzing the SSI response of a typical nuclear reactor building in both frequency and time domain. The total SSI system was first analyzed in frequency domain using SASSI [12] to obtain the baseline solution. Following this, the foundation was modeled as a rigid massless cavity (without structure) and used to derive the scattered motions and DFI discrete functions, which was then linearized to obtain DPFI model. Finally, the scattered motions and DPFI model were incorporated into the ADINA model of the structure, which was then solved in time domain using direct integration method. The results of ADINA were then compared with those of baseline solution obtained from one-step SASSI analyses.

Structural Model

The validation model is shown in Fig. 3. The model represents a typical PWR containment structure consisting of a primary concrete containment, a steel secondary containment and concrete internal structures housing the reactor. This model is an idealization of a typical pressurized water reactor and does not represent an actual or existing structure. The cylindrical reactor cavity is deeply embedded below ground surface. For simplification, the steel containment and parts of the internal structure such as the steam generator and pressurizer compartments are not modeled. Floor slabs are simplified and included mainly for their effect on the structural response. The concrete containment cylinder wall and dome are 0.90m and 0.75m thick, respectively. The internal structures consist of a 0.60m-thick cylindrical wall and reactor cavity/fuel pool structure coupled through the floor slabs. The foundation base slab is 7.00m thick and embedded approximately 13.50m below grade. The reactor vessel mass is included as lumped mass in the model. The concrete containment and internal cylindrical walls are modeled with plate/shell elements. The foundation slab/side walls and reactor cavity are modeled with solid elements. The SASSI and ADINA structural models are comparable in terms of finite elements, and node and element numbering system. In SASSI model, the excavated soils are modeled using solid elements. In ADINA model, the foundation stiffness, mass and damping (KMC) is modeled using general matrix elements. A 7% uniform damping is used for all structural elements. For ADINA, the Rayleigh damping parameters were calculated such that the damping does not exceed 7% at frequencies between 4 and 16 Hz for the containment, and between 4 and 13 Hz for the internal structures. This ensures conservative damping for the containment and internal structures modes, as shown in Table 1.

In terms of model size, the structure has 10,002 nodes of which 1,805 are interaction nodes located at soil/structure interface. The SASSI model has an additional 8,239 excavated soil nodes for a total model size of 18,241 nodes.

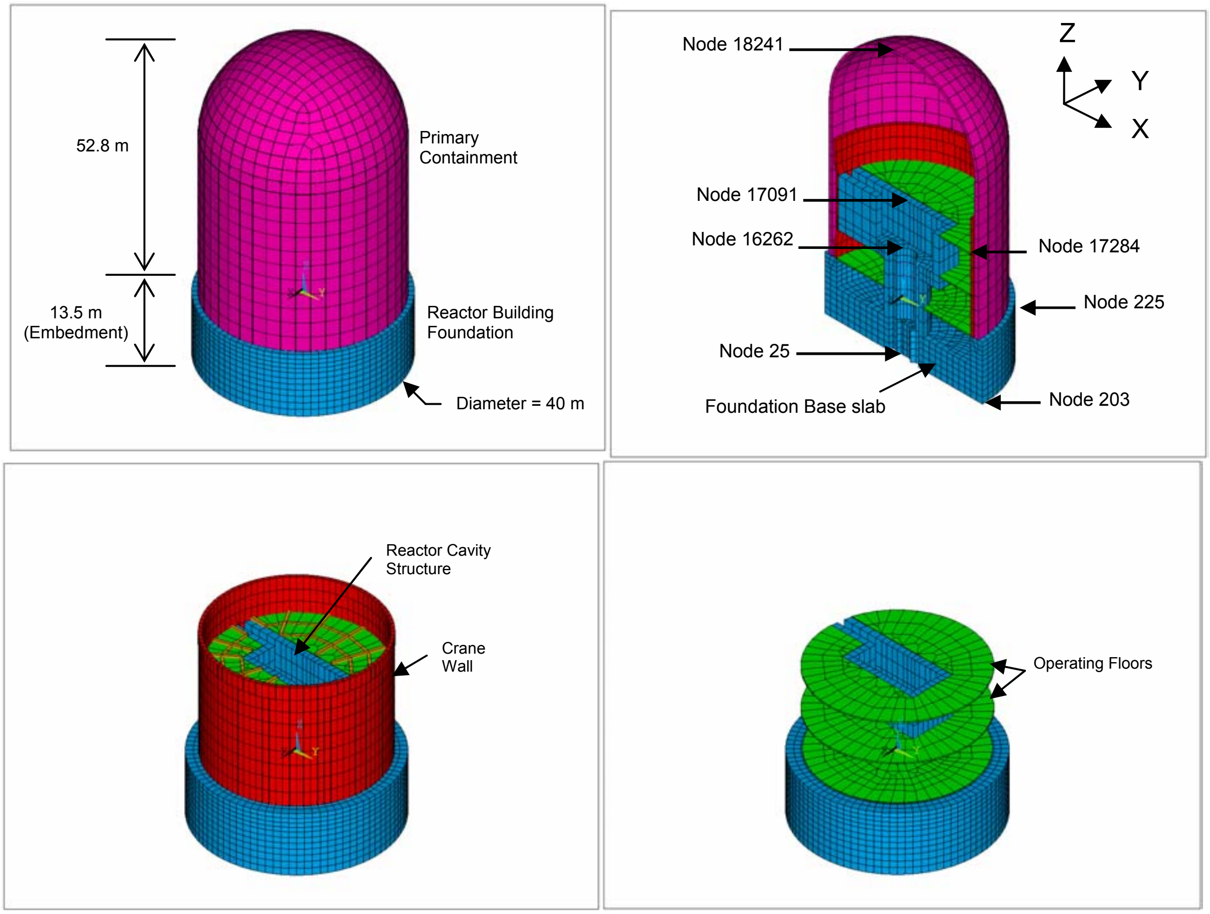


Fig. 3– SASSI/ADINA Finite Element Structural Models

Ground Motions and Site Response

Ground motions selected for this study consist of two horizontal acceleration time histories (used as x and y input motions) recorded at El Centro Array #9 during 1940 Imperial Valley earthquake in California. These acceleration time histories and their 5%-damped acceleration response spectra (ARS) are shown in Fig. 4. The ground motions correspond to the free-field surface motion of a stiff soil profile (Vs=500 m/s) and are assigned peak ground acceleration of 0.50g and 0.34g in x and y directions, respectively.

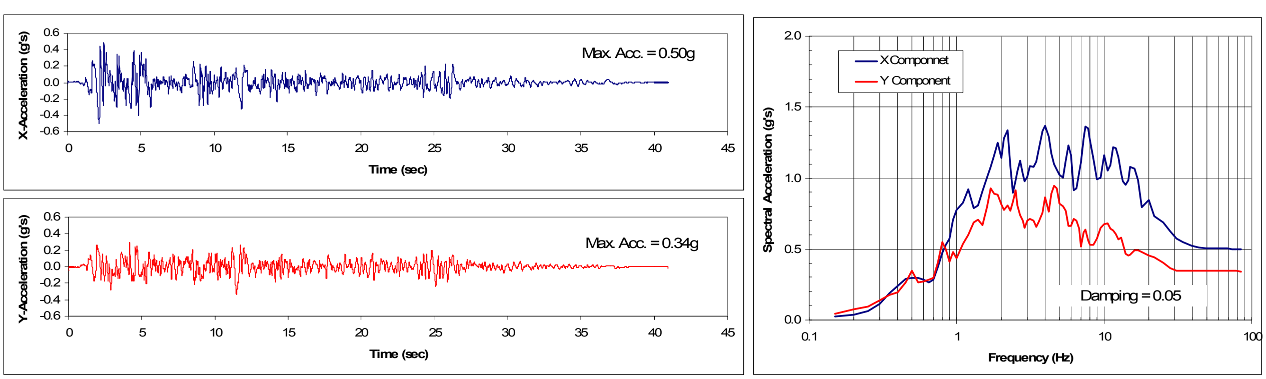


Fig. 4 – Acceleration Time Histories and ARS of Free Field Horizontal Input Motions

The assumed subsurface profile consists of a deep stratum of dense to very dense granular soil. Figure 5 shows the density and low-strain velocity profile. One-dimensional site response SHAKE [13] analyses were performed using the above motions specified at the free-field ground surface to develop strain-compatible soil shear wave velocity and damping ratio, as shown in Fig. 5. This figure also shows the variation of peak ground acceleration versus depth computed from SHAKE analysis. The above soil profile and strain-compatible soil properties were utilized in the subsequent SSI analyses.

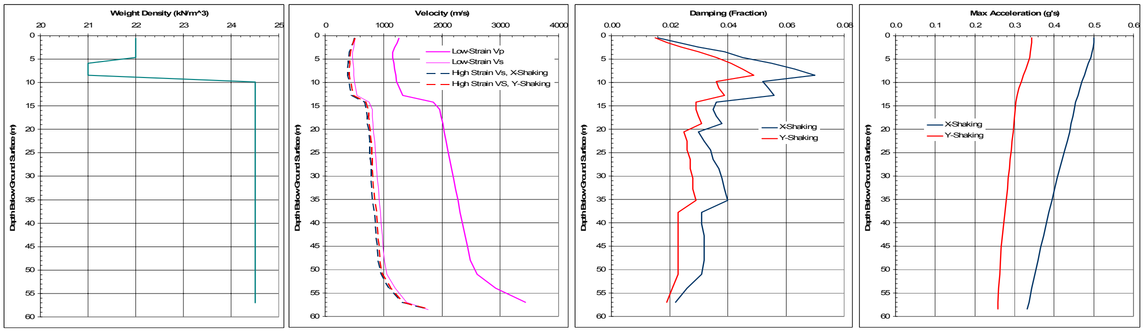


Fig. 5 –Soil Profile and Properties

Fixed-Base Analyses

Fixed-base analyses of the reactor building were performed using both SASSI (frequency domain) and ADINA (time domain) to verify the accuracy of finite element models and analysis procedure used in these programs. In both models, the base slab was fixed in translations and was subjected to horizontal input in the x and y directions. Table 1 lists the first few un-damped natural frequencies of the structure obtained from ADINA modal analyses and SASSI transfer functions. In addition, the horizontal spectral acceleration response of the structure at top of primary containment (Node 18241), top of internal structures (Node 17091), corner of floor slab/shield wall (Node 17284) and base slab (Node 25) subject to scattered input motion in the x and y direction were calculated and are compared in Figure 6 and 7, respectively. The scattered motions were obtained from the scattering analyses, as discussed later. These results show excellent agreement between SASSI and ADINA fixed-base analysis results verifying the accuracy of the two structural models and dynamic analysis procedures.

Table 1 – Undamped Fixed-Base Natural Frequencies of Reactor Building (Hz)

Mode	ADINA	SASSI
Primary Containment, 1st Mode, X-Dir.	5.26	5.21
Primary Containment, 1st Mode, Y-Dir.	5.26	5.21
Internal Structures, 1st Mode, X-Dir.	8.55	8.37
Internal Structures, 1st Mode, Y-Dir.	8.82	8.69
Primary Containment, 1st Mode, Z-Dir.	14.56	14.40
Primary Containment, 2nd Mode, X-Dir.	15.53	15.41
Primary Containment, 2nd Mode, Y-Dir.	15.59	15.49

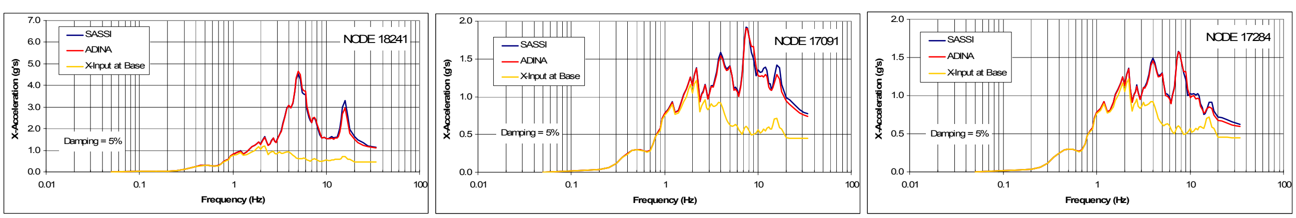


Fig. 6 – Comparison of Fixed-Base In-Structure ARS Due to Input Motion in X-Dir

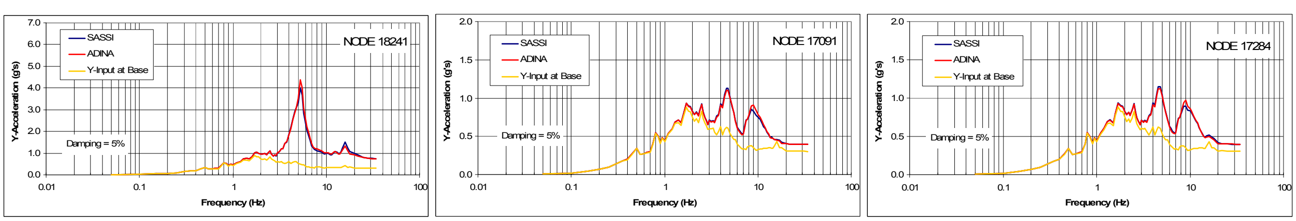


Fig. 7 – Comparison of Fixed-Base In-Structure ARS Due to Input Motion in Y-Dir

Soil-Structure Interaction Analyses

Baseline Solution: One-step analysis of the total SSI system of reactor building/foundation model was performed using SASSI. The structure and soil model are shown in Figure 3 and 5, respectively. The SSI model was subjected separately to input motion in the x and y directions, as shown in Fig. 4. The in-structure spectral acceleration response of the containment and internal structures were calculated at nodes 18241, 16262, 17091 and 17284 (see Fig. 3). In addition, the horizontal and rocking response of base slab were also calculated from the horizontal response at center of base slab (Node 25) and vertical response at corner of base slab (Node 203/225), respectively. The results obtained at top of structures include the effect of phasing on coupled horizontal and rocking response. The results of one-step SASSI analyses are presented in Fig. 10 and 11 for the x and y input, respectively, and considered the baseline solutions for validation purposes.

Foundation Impedance and Scattering Analyses: The subgrade dynamic impedance obtained from baseline analysis was used to restart SASSI with a new structural model consisting of the foundation cavity modeled with a rigid massless cylindrical shell, as shown in Fig. 8. This model was subjected separately to input motion in the x and y directions to compute the respective foundation scattering motions. Following this, the same model was subjected to unit amplitude harmonic forces applied in the x, y and z directions at the bottom center of base slab and the resulting forces and displacements at all foundation interaction nodes were calculated and used to calculate DFI functions.

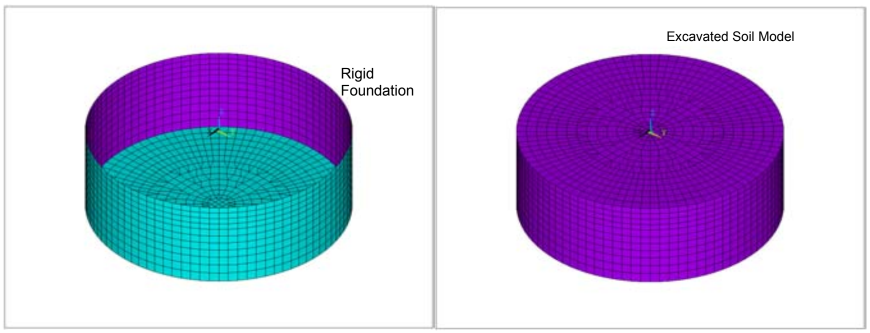


Fig. 8 – SASSI Finite Element Models for Impedance/Scattering Analysis

DPFI Model Development and Validation: The distributed, uncoupled, frequency-dependent DFI functions developed above were linearized based on response of a simple damped oscillator to calculate DPFI model in terms of constant spring, mass and dashpot parameters (KMC). The linearization was based on least square method fit in the frequency range 0-10 Hz for all three directions. The DPFI model was then incorporated as boundary condition in the structural model of reactor building subjected to scattered motions developed above and analyzed in frequency domain using SASSI to obtain the response of the system. The results were compared with those of baseline solution for x and y input, respectively, to check the accuracy of impedance linearization.

Time Domain Analyses: The DPFI model and scattered motions were then incorporated in the structural model of reactor building and analyzed in time domain by ADINA using step-by-step integration method. The implementation of KMC consisted of first defining a set of ground nodes coincident with the interaction nodes and then coupling each set of interaction/ground nodes with a mass, dashpot and spring matrix element with uncoupled parameters in the x, y and z directions. The scattered motions in terms of displacement time histories were then imposed at all ground nodes. Because scattered motions represent the response of a rigid cavity, their incorporation in the ADINA model was greatly simplified by only specifying the translational and rotational scattered motions at a separate node coincident with the center of base slab (Node 25) and using constrained equations to couple it to all ground nodes. The results of the ADINA time-domain analyses are compared with those of baseline solution in Fig. 10 and 11 for x- and y-input, respectively.

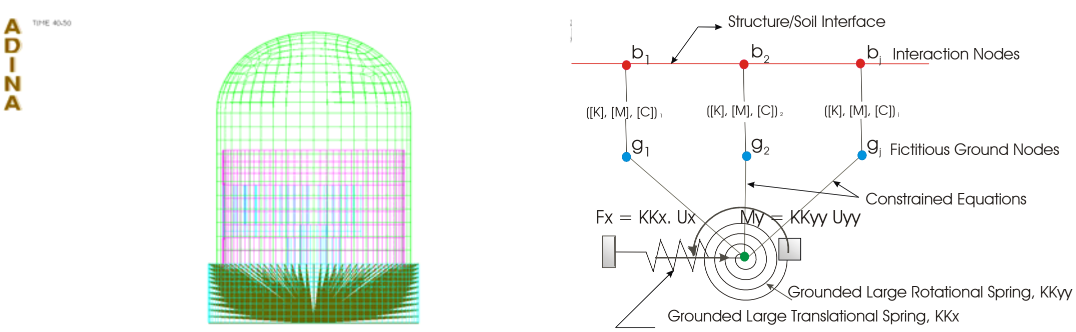


Fig. 9 – Schematic View of Application of DPFI Model and Scattered Motions in ADINA Structural Model

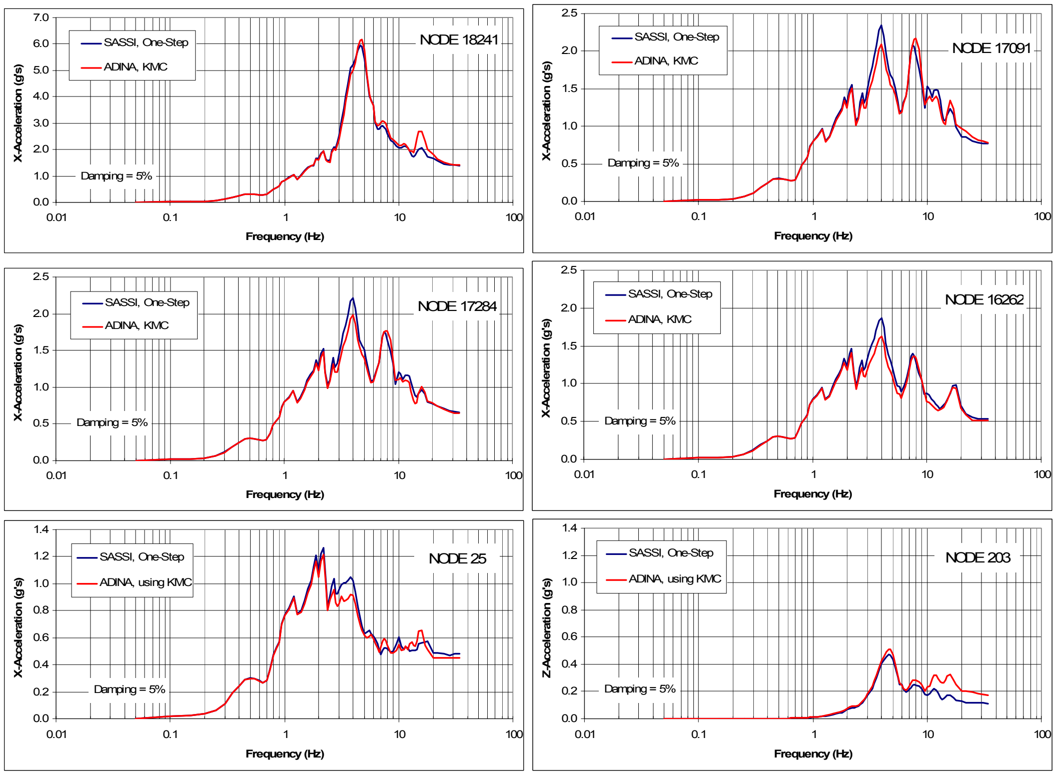


Fig. 10 – Comparison of SSI In-Structure ARS Due to Input Motion in X-Dir

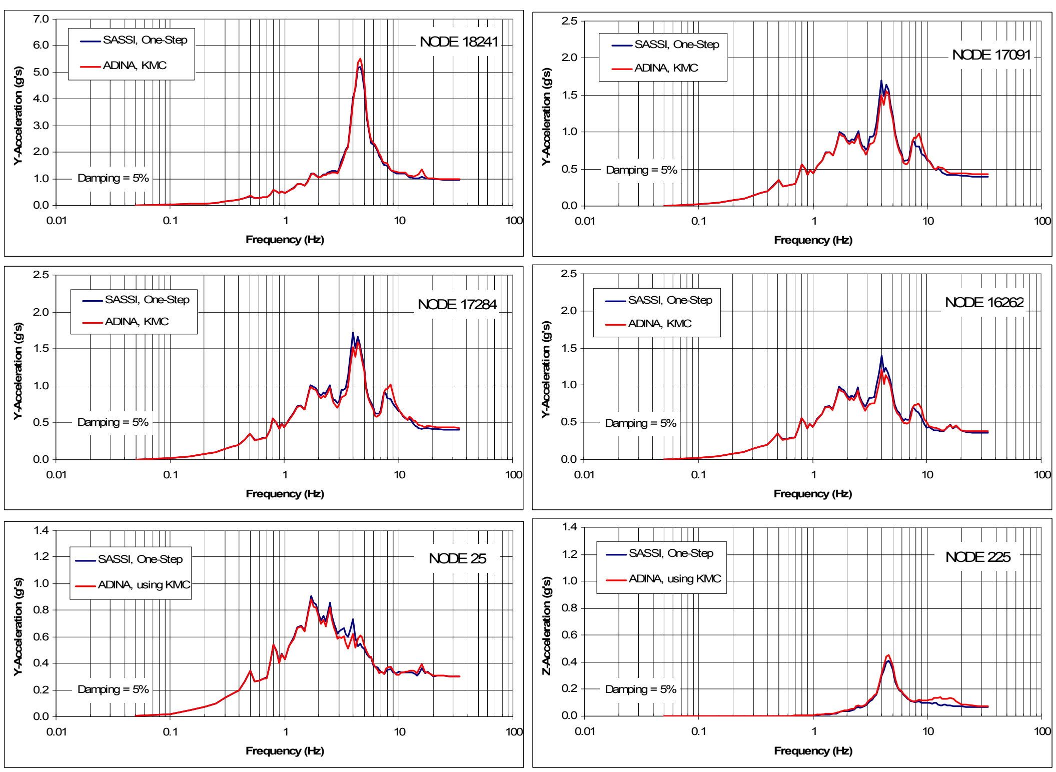


Fig. 11 – Comparison of SSI In-Structure ARS Due to Input Motion in Y-Dir

Discussion of Results

Comparison of in-structure response of primary containment and internal structures, as shown in Fig. 10 and 11 show reasonably good agreement between SASSI one-step baseline solution (frequency-domain) and ADINA (time domain) using DPFI model. The maximum acceleration, peak spectral response and peak spectral frequencies are captured within reasonable accuracy at all nodes, as compared above from ADINA analyses using DPFI model. Some higher rocking of base slab calculated from ADINA at frequencies above 10 Hz (see Nodes 203 and 225 in Fig. 10 and 11, respectively) as compared to the baseline solution is attributed to frequency fit of KMC between 0-10 Hz.

SUMMARY AND CONCLUSIONS

An improved method for SSI analysis in time domain using distributed parameter foundation impedance (DPFI) model derived from SASSI was presented. The methodology consists of three steps: first the foundation reaction forces and interaction displacements are calculated from analyses of the total SSI system in frequency domain and used to calculate uncoupled, frequency-dependent distributed foundation impedance (DFI) functions, second the DFI functions are linearized based on the response of a simple damped oscillator using least square technique on a selected frequency range to obtain a set of distributed KMC parameters (DPFI model) and finally, DPFI model is implemented in time domain analyses of the structure (without soil model) using kinematic interaction formulation.

The accuracy of the above procedure for an embedded structure with very stiff/rigid basement is validated by comparing the SSI response of a typical PWR containment building subject to horizontal input motions obtained from one-step analyses of the total SSI system in frequency domain using SASSI (baseline solution) against those of ADINA analysis with DPFI model in time domain using direct integration method. The reasonably good agreement between the SASSI and ADINA in-structure results demonstrates the effectiveness of the procedure presented herein.

It is noted that the quality of the results obtained from time-domain analyses depends on the frequency range and order of foundation impedance fits. The use of single oscillator model to represent impedance functions is found to be adequate for the reactor model analyzed herein. Nonetheless, higher order frequency functions that utilize multiple oscillators in parallel or series are also available that can provide better fit to foundation impedances. Multiple oscillator systems are, in general, more complex and may require specialized optimization methods to derive their properties.

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