DISTRIBUTED PARAMETER FOUNDATION IMPEDANCE MODEL FOR TIME DOMAIN SSI ANALYSIS

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ABSTRACT

In this paper we present a methodology for developing distributed parameter foundation impedance (DPFI) model for time-domain soil-structure interaction (SSI) analysis of structures supported on large flexible mat foundations at ground surface. This development is performed in three steps. First the foundation displacements and reaction forces in the x, y and z directions at each foundation interaction DOF are calculated from analysis of the total SSI system in frequency domain, and used to form the distributed foundation impedance (DFI) matrix. In the second step, the resulting uncoupled, frequency-dependent DFI matrix is linearized using a spring and a dashpot attached in parallel to a mass to constitute a simple damped oscillator at each interaction DOF. In the final step the DPFI (spring-mass-dashpot) model is implemented in time-domain analysis of the structure using kinematic or inertial interaction formulation. The effectiveness of this procedure is demonstrated by analyzing the frequency response of a lumped mass parameter model supported on rigid and flexible foundation mat subject to horizontal and vertical excitations. A comparison of the results obtained from direct analysis of the total SSI system in frequency domain and SSI analysis of the structure in time domain using DPFI model show good agreement for both cases of horizontal and vertical excitations.

Introduction

The desire to break a complex problem into smaller more manageable systems has led to development of substructuring technique for dynamic response analyses of large SSI systems (Kausel, et. al. 1978, and Lysmer and Tabatabaie, et. al. 1991). In substructuring method, the total SSI system is partitioned into two subsystems, namely the structure and foundation. The foundation is analyzed first, generally in frequency domain, and the foundation dynamic impedance and scattering properties are established at the structure/foundation interface. These properties are then used as boundary condition in dynamic analysis of the structure, which is generally performed in time domain. This approach offers several advantages in practice. For example from the foundation point of view, it enables the analyst to properly a) address the effects of wave radiation in unbounded soil media, b) incorporate strain-compatible soil modulus and damping properties and c) specify input motion in the free field using deconvolution method. From the structural point of view, because the analysis is performed in time domain, the analyst can consider for example nonlinear behavior of the structure using time history dynamic response analysis. Nonetheless, because two different dynamic solution methods (frequency versus time domain) are mixed to analyze the SSI system, significant difficulties often arises

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when complex frequency-dependent impedance and scattering properties derived from analysis of the foundation in frequency domain are to be incorporated as boundary condition in time domain analyses of the structure (so called "handshake").

For structures supported on rigid mats, the foundation impedance and scattering properties are greatly simplified, i.e. the impedance matrix is reduced to a 6x6 matrix and the scattering problem becomes the free field problem. This class of problems has been extensively studied by other researchers (Wolf 1991, 1994, 1997; and De Barros and Luco 1990 among others) and methods using lumped parameter foundation impedance (also referred to as constant spring-mass-dashpot) models have been devised to address the frequency/time-domain handshake. For flexible mats, development of foundation dynamic impedance is more complex than that of rigid mats. This is mainly due to the fact that the response of a flexible mat cannot be fully described by three translations and three rotations at the center of mat. When the structure is connected to the flexible mat, each point on the mat moves differently, and it is not possible to develop one lumped parameter model to represent the entire soil-flexible mat foundation system for each mode of vibration. A relatively large dynamic impedance matrix incorporating many nodes on the mat is required to accurately capture the flexible behaviour of the mat.

In the following we present a methodology for developing distributed foundation impedance (DFI) model for a flexible mat foundation supported at the ground surface. It provides for two horizontal (x and y) and a vertical (z) component at each foundation interaction DOF similar to soil springs attached to the bottom of the mat. Rotational components are not defined in this case and the rotational behaviour of the mat is governed by the vertical impedance. The DFI is then linearized to develop distributed parameter foundation impedance (DPFI) model for implementation in time domain analyses. In this case, the scattering problem need not be solved as it reduces to the free field problem.

Distributed Foundation Impedance Formulation

The general linear dynamic equation of motion for a structure supported on a flexible mat at ground surface and subject to a seismic and/or dynamic input (see Fig. 1) may be written in frequency domain as follows.

$$(-\omega^2 \underline{M} + i\,\omega\,\underline{C} + \underline{K}) \cdot \underline{U} = \underline{Q} \tag{1}$$

Where $\underline{M}$, $\underline{C}$ and $\underline{K}$ are the total mass, damping and stiffness matrices, respectively, $\underline{Q}$ is the vector of external forces, $\underline{U}$ is the total displacement response vector, ω is circular frequency and $i=\sqrt{-1}$. For brevity in all later discussions all underscores for matrix or vector representation are dropped and the complex coefficient matrix in Eq. 1 is replaced by $K = (-\omega^2 \underline{M} + i\,\omega\,\underline{C} + \underline{K})$.

The total SSI system shown in Fig. 1 may be partitioned into two substructures, namely the "foundation" and "structure", as shown in Fig. 1(a) and 1(b), respectively. The structure consists of the structure plus flexible mat and the foundation consists of soil media. The interaction between the two substructures occurs at three translational DOFs of all mat nodes connected to the ground. For the above substructuring, the equation of motion for the structure and foundation may be written as shown in Eq. 2 and 3, respectively.

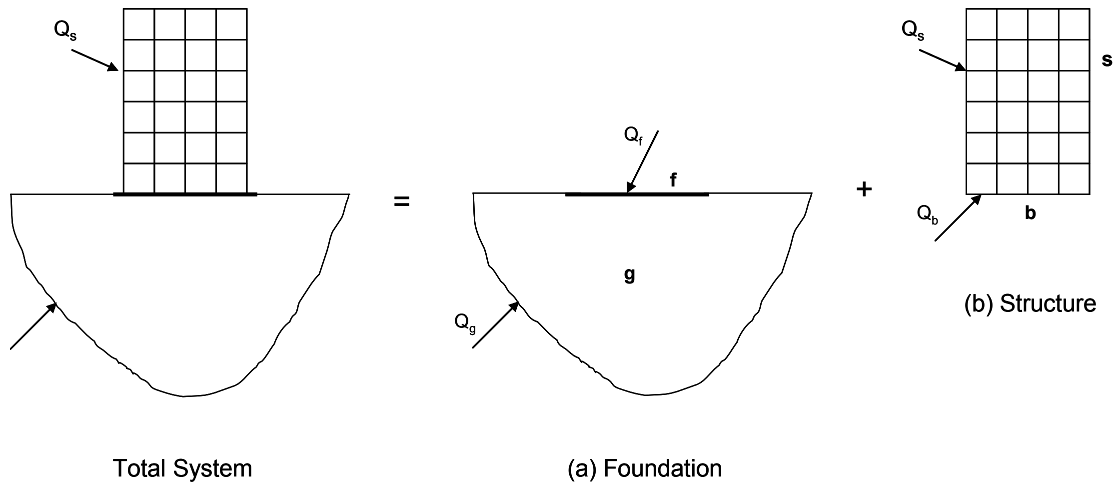


Figure 1. SSI substructuring method

$$\begin{pmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb} \end{pmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} Q_s \\ Q_b \end{Bmatrix} \tag{2}$$

$$\begin{pmatrix} K_{ff} & K_{fg} \\ K_{gf} & K_{gg} \end{pmatrix} \begin{Bmatrix} U_f \\ U_g \end{Bmatrix} = \begin{Bmatrix} Q_f \\ Q_g \end{Bmatrix} \tag{3}$$

Where “s”, “b”, “f” and “g” denote the structure, mat, interaction and ground DOFs, respectively. U and Q are the total displacement and boundary forces, respectively.

When the structure is non-existent (i.e. $Q_f = 0$), Eq. 3 reduces to that of the free field problem, as shown in Eq. 4.

$$\begin{pmatrix} K_{ff} & K_{fg} \\ K_{gf} & K_{gg} \end{pmatrix} \begin{Bmatrix} U^*_f \\ U^*_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_g \end{Bmatrix} \tag{4}$$

Where U^* is now the free field ground motions. Subtracting Eq. 4 from Eq. 3 results in Eq. 5,

$$\begin{pmatrix} K_{ff} & K_{fg} \\ K_{gf} & K_{gg} \end{pmatrix} \begin{Bmatrix} U_f - U^*_f \\ U_g - U^*_g \end{Bmatrix} = \begin{Bmatrix} Q_f \\ 0 \end{Bmatrix} \tag{5}$$

Condensing $U_g - U^*_g$ from Eq. 4 results in Eq. 6.

$$X_f \,.\, R_f = Q_f \tag{6}$$

Where $R_f = U_f - U^*_f$ represents the interaction displacements, Q_f is the vector of soil reaction forces and X_f is the subgrade dynamic impedance. X_f is a full and complex matrix.

Imposing the boundary conditions between the two substructures (U_b=U_f and Q_b+Q_f=0) and Substituting Q_f from Eq. 5 into Eq. 4 and rearranging the terms results in Eq. 7.

$$\begin{pmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb}+X_f \end{pmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ X_f \, . \, U^*_f \end{Bmatrix} \tag{7}$$

Equation 7 represents the general response of the total SSI system shown in Fig. 1. To solve Eq. 7 for the structure, the subgrade dynamic impedance (X_f) and free field ground motions (U^*_f) at all soil-mat interaction DOFs are needed.

Assuming that the foundation reaction forces, Q_f, and interaction displacements, R_f, are known, Eq. 6 may be rearranged, as follows:

$$X_f \, . \, R_f = Q_f = [q_j/r_j]_f \, . \, R_f = D_f \, . \, R_f \tag{8}$$

Where D_f =$[q_j/r_j]_f$ is a diagonal matrix representing the complex stiffness of foundation interaction DOFs; and q_j and r_j are the soil reaction force and displacement at interaction DOF j.

Substituting D_f for X_f from Eq. 8 into Eq. 7, we obtain:

$$\begin{pmatrix} K_{ss} & K_{sb} \\ K_{bs} & K_{bb}+D_f \end{pmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ D_f \, . \, U^*_f \end{Bmatrix} \tag{9}$$

As can be seen above, Eq. 9 is the same as Eq. 7 except that the full subgrade impedance matrix has now been replaced by a diagonal foundation impedance matrix consisting of uncoupled springs. Equation 9 offers an advantage over Eq. 7 in that D_f does not contain any off-diagonal terms, i.e. it can be used to develop equivalent foundation springs for time domain analysis. It should be noted that Eq. 9 will provide exact solution to the SSI problem, as shown in Fig. 1, only if the soil reaction forces, Q_f, and interaction displacements, R_f, are known. Because Q_f and R_f depend on the foundation configuration and dynamic loading, D_f is referred to as "distributed foundation impedance (DFI)".

The DFI is calculated by first solving the total SSI system in frequency domain to obtain $Q_f = \{q_j\}_f$ and R_f, = $\{r_j\}_f$ at each interaction DOF j and then substituting them in Eq. 8 to calculate D_f.

Distributed Parameter Foundation Impedance Formulation

In general, the DFI matrix, D_f, is complex and frequency dependent. Each element of DFI, d_j, constitutes a complex frequency-dependent function representing dynamic impedance at an interaction DOF. In addition, it can be shown that the real part of the dynamic impedance is associated with foundation stiffness while the imaginary part represents foundation damping.

Because frequency-dependent foundation stiffness and damping parameters can not be directly used in time domain analyses, they must be linearized using single (or multiple) damped oscillator system having constant spring (k_j), dashpot (c_j) and mass (m_j) properties. The resulting system is referred to as "distributed parameter foundation impedance (DPFI)" model.

In this paper we will only present results of linearization using single damped oscillator system, as shown in Fig. 2. The complex stiffness of a single damped oscillator with constant parameters (k_j-c_j-m_j) subject to a dynamic force is illustrated in Fig. 2 and presented in Eq. 12.

$$K_j(\omega) = - \omega^2 . m_j + i \omega c_j + k_j \tag{12}$$

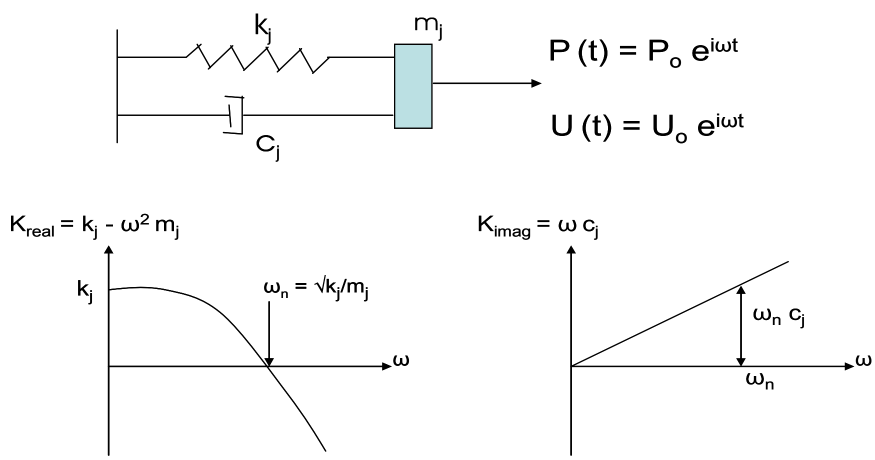


Figure 2. Representation of foundation impedance with single damped oscillator

By representing foundation impedance as a single damped oscillator, we can write:

$$\text{Real}(d_j) = \text{Real}(K_j) = k_j - \omega^2 . m_j \tag{13}$$
$$\text{Imag}(d_j) = \text{Imag}(K_j) = \omega c_j \tag{14}$$

Equations 13 and 14 form the basis for estimating the single oscillator properties to represent frequency response behaviour of the foundation stiffness and damping. This is done by first setting k_j to the static stiffness, i.e. k_j=Real[$d_j(\omega=0)$], and then calculating m_j and c_j based on least square method of minimizing error over the frequency range of interest. The resulting mass component of the oscillator, m_j is, often, referred to as "virtual foundation mass".

The use of single oscillator model to represent impedance functions, in general, is adequate for dynamic systems whose frequency response of interest is dominated by a single mode. For higher order impedance functions, often a system of 3 to 5 oscillators in parallel or series are required to adequately fit the frequency function. Multiple oscillator systems are, in general, more complex and may require specialized optimization methods to derive their properties.

Implementation of DPFI Model in Time Domain SSI Analysis

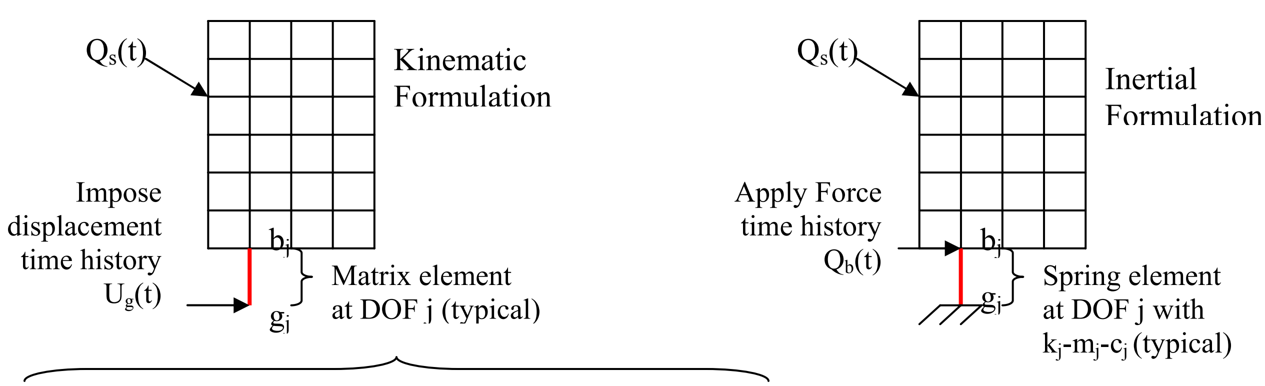
DPFI model may be implemented in time domain SSI analysis using either kinematic or inertial formulation. In kinematic formulation, the oscillators are placed between the structure and ground interaction DOFs, and the input displacement time histories are imposed at all ground DOF, as shown in Fig. 3 and Eq. 15. It is noted that in the kinematic formulation, the virtual foundation mass, m_g, should be included in the coefficient matrix with negative off-diagonal terms and not simply as lumped masses. By condensing the last equation out of Eq. 15, the inertial formulation of the equation of motion is obtained, as illustrated in Fig. 3 and Eq. 16. In the inertial formulation, the ground interaction DOFs are fixed and the input motion is applied as external forces, $F_b(t)$ to the structure at all interaction DOFs, as shown in Eq. 16. As seen from Eq. 16, inertial formulation eliminates the need for using negative off-diagonal foundation mass in the coefficient matrix.

$$\begin{pmatrix} M_s & 0 & 0 \\ 0 & M_b+m_g & -m_g \\ 0 & -m_g & m_g \end{pmatrix} \begin{Bmatrix} U_s''(t) \\ U_b''(t) \\ U_g''(t) \end{Bmatrix} + \begin{pmatrix} C_{ss} & -C_{sb} & 0 \\ -C_{bs} & C_{bb}+c_g & -c_g \\ 0 & -c_g & c_g \end{pmatrix} \begin{Bmatrix} U_s'(t) \\ U_b'(t) \\ U_g'(t) \end{Bmatrix} +$$

$$\begin{pmatrix} K_{ss} & -K_{sb} & 0 \\ -K_{bs} & K_{bb}+k_g & -k_g \\ 0 & -k_g & k_g \end{pmatrix} \begin{Bmatrix} U_s(t) \\ U_b(t) \\ U_g(t) \end{Bmatrix} = \begin{Bmatrix} Q_s(t) \\ 0 \\ Q_g(t) \end{Bmatrix} \tag{15}$$

$$\begin{pmatrix} M_s & 0 \\ 0 & M_b+m_g \end{pmatrix} \begin{Bmatrix} U_s''(t) \\ U_b''(t) \end{Bmatrix} + \begin{pmatrix} C_s & -C_b \\ -C_b & C_b+c_g \end{pmatrix} \begin{Bmatrix} U'_s(t) \\ U'_b(t) \end{Bmatrix} + \begin{pmatrix} K_s & -K_b \\ -K_b & K_b+k_g \end{pmatrix} \begin{Bmatrix} U_s(t) \\ U_b(t) \end{Bmatrix} = \begin{Bmatrix} Q_s(t) \\ Q_b(t) \end{Bmatrix} \tag{16}$$

Where $Q_b(t) = m_g.U_g''(t)+ c_g.U_g'(t)+ k_g.U_g(t)$, and $U''(t)$, $U'(t)$ and $U(t)$ are acceleration, velocity and displacement time histories.



$$k_{g,j} = \begin{pmatrix} k_j & -k_j \\ -k_j & k_j \end{pmatrix} \quad m_{g,j} = \begin{pmatrix} m_j & -m_j \\ -m_j & m_j \end{pmatrix} \quad c_{g,j} = \begin{pmatrix} c_j & -c_j \\ -c_j & c_j \end{pmatrix}$$

Figure 3. Implementation of DPFI in time domain model

Validation Problem

To demonstrate the accuracy of DPFI model, the frequency response of a lumped mass parameter model supported on a square mat foundation on uniform halfspace is examined. The dynamic properties of the structure, foundation mat and soil media are summarized in Fig. 4. The model is subjected separately to vertical and horizontal excitation by respectively applying vertically propagating harmonic P- and SV-waves with control motion specified at the free-field ground surface.

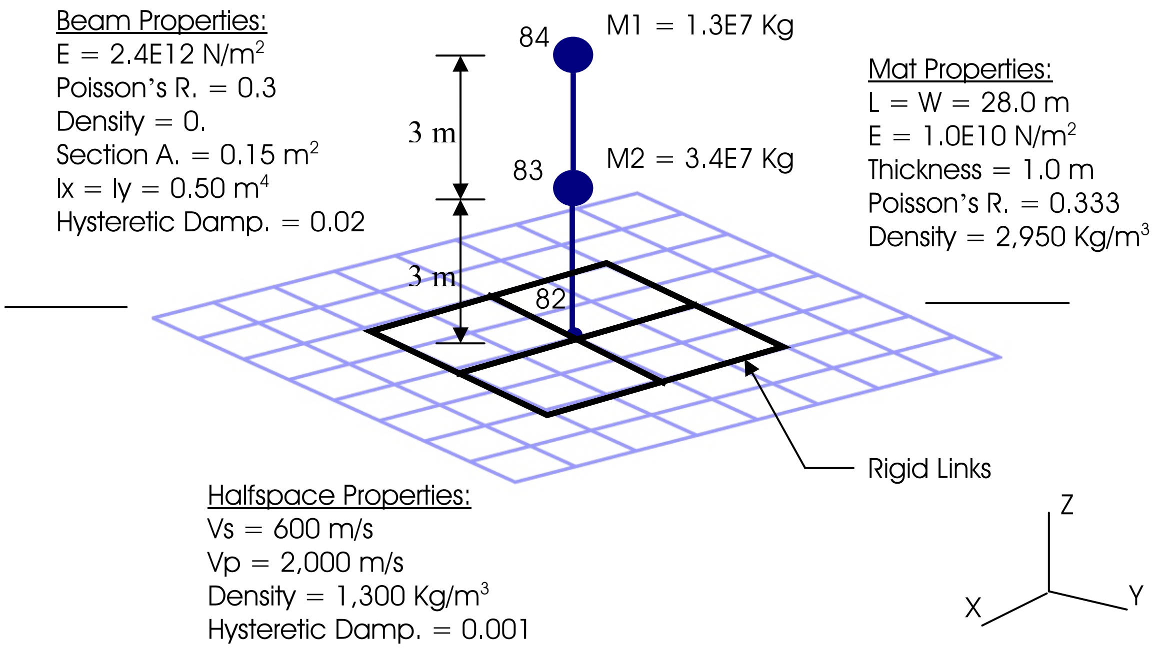


Figure 4. Lumped mass parameter test model

The total SSI system was first analyzed in frequency domain using SASSI (Lysmer et. al. 1981 and MTR/SASSI 2004) to obtain the baseline solution. Both cases of rigid and flexible mat foundation were considered. Figure 5 and 6 show the vertical response of Node 84 from vertical excitation for cases of rigid and flexible mat, respectively. Similarly, the horizontal response of Node 84 obtained from horizontal excitation is shown in Figs. 7 and 8 for rigid and flexible mat, respectively. The results are presented in terms of transfer function relative to the control motion specified at free field ground surface.

Following this, the response of the structure was calculated in frequency domain with SASSI using DFI as well as DPFI model. A total of 81x3=243 DFI functions were calculated from direct solution of the SSI model, as described above. Each of these functions was then fit by response of a single damped oscillator with constant k_i-m_i-c_i parameters within a frequency range of 0-6 Hertz. Other frequency ranges were also examined but did not provide significant improvement of the results. The results of both analyses are compared with those of the baseline solution in Figs. 5 and 6 for vertical and in Figs. 7 and 8 for horizontal excitation, respectively.

Finally, the LMP model was analyzed in time domain with ADINA (ADINA 2004) using the above DPFI (k_i-m_i-c_i) model. The time domain analyses were performed using displacement input time histories and step-by-step integration using Newmark integration method. In ADINA model 2% Rayleigh damping was anchored to frequencies of 4.2 and 18.2 Hertz for vertical analysis and 3.5 and 11.0 Hertz for horizontal analysis of both rigid and flexible mats, respectively. The results of the time-domain analysis are compared with the baseline results in Figs. 5, 6, 7 and 8.

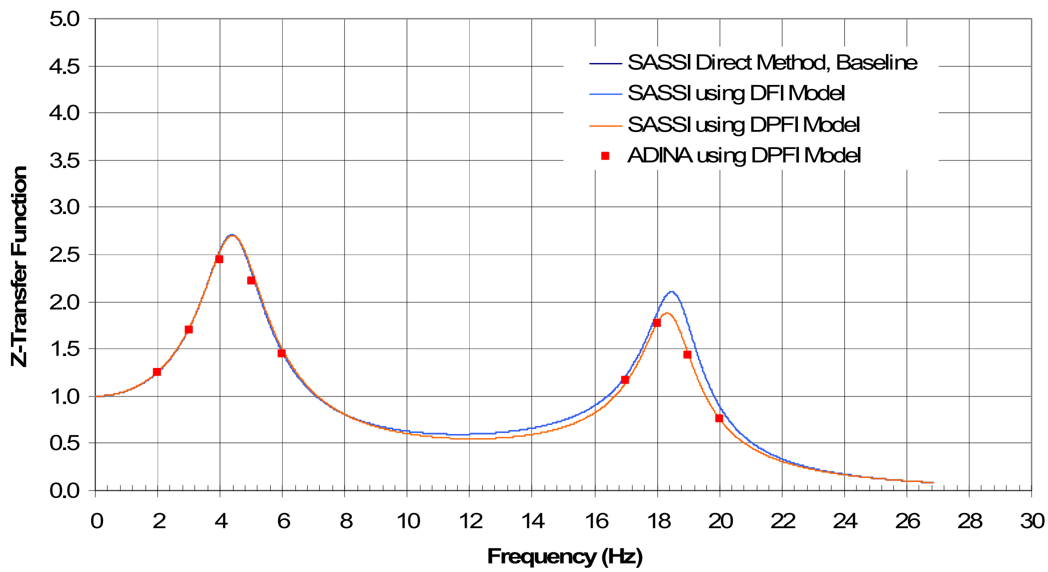


Figure 5. Vertical transfer function at Node 84, vertical excitation, rigid mat case

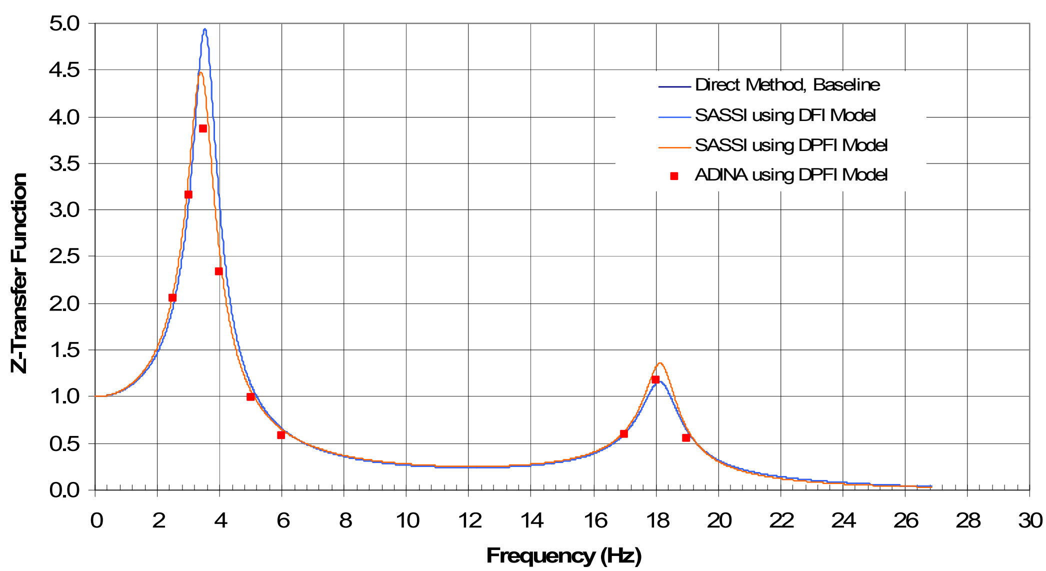


Figure 6. Vertical transfer function at Node 84, vertical excitation, flexible mat case

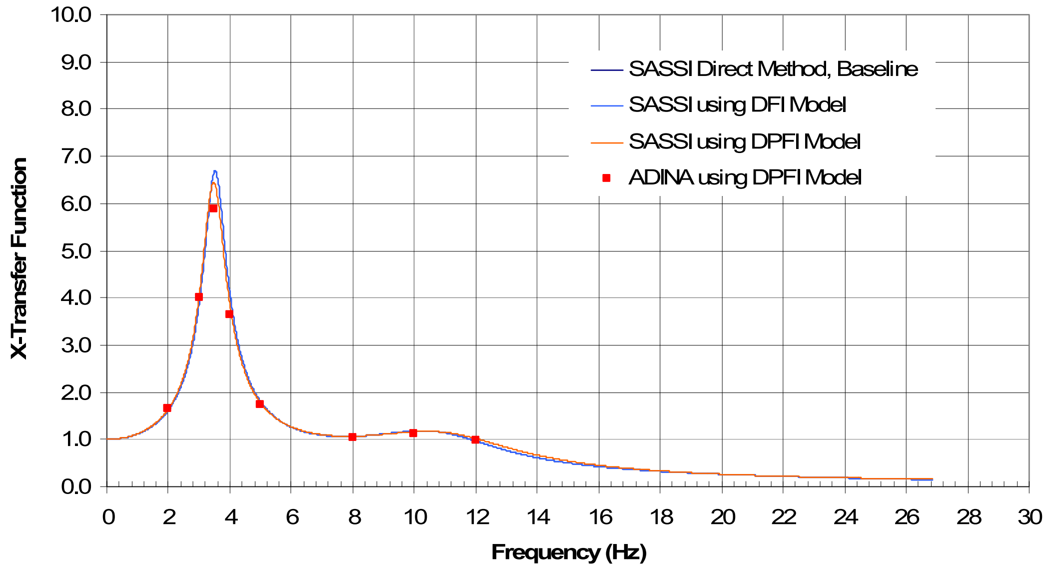


Figure 7. Horizontal transfer function at Node 84, horizontal excitation, rigid mat case

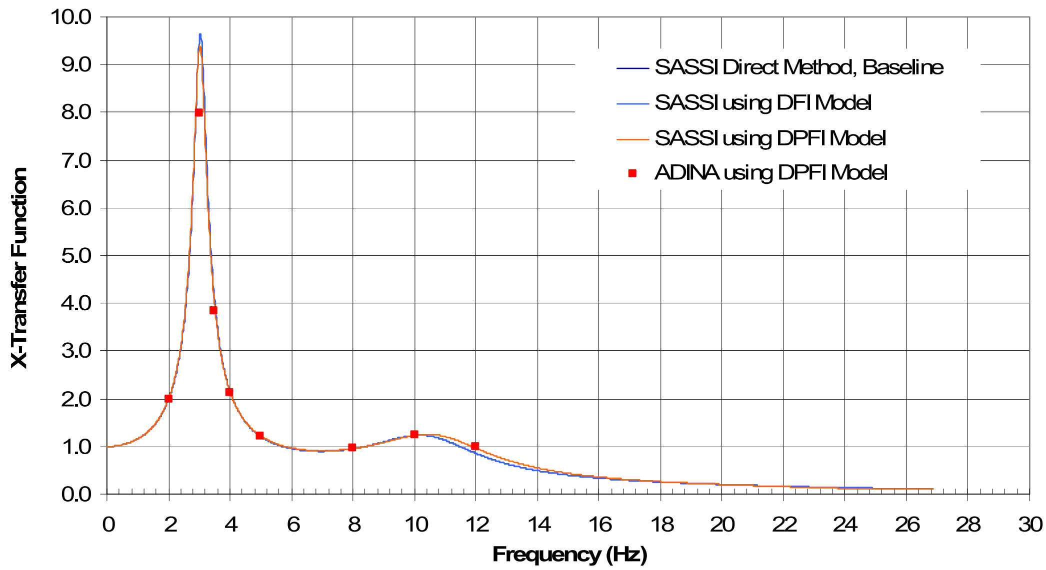


Figure 8. Horizontal transfer function at Node 84, horizontal excitation, flexible mat case

As seen from the above figures, for both rigid and flexible mat cases and both vertical and horizontal excitation, the response of the structure computed using DFI model are the same as those of the direct analysis of the total SSI system. This verifies the accuracy of the methodology for deriving uncoupled frequency-dependent DFI model. The above figures show excellent agreement between SASSI and ADINA solutions for both flexible and rigid mat cases using DPFI model. This verifies implementation of DPFI model in time domain analysis. The results using DPFI model are also within 10% of the baseline solution obtained using direct SASSI analyses demonstrating accuracy of the linearization scheme.

Conclusions

A simplified method for developing distributed parameter foundation impedance (DPFI) for use in time domain analysis of flexible mat foundations supported at ground surface was presented. It was shown from the theoretical formulations and validation problem that when the soil reaction forces and interaction displacements are accurately determined from analysis of the total SSI system in frequency domain and used to derive the distributed foundation impedance (DFI), the computed response of the structure on flexible mat computed from substructuring analysis in frequency domain exactly match those of the direct analysis of the total system. It is further shown that when the above frequency-dependent DFI functions are used to derive distributed parameter foundation impedance with constant parameters (spring-dashpot-mass) for use in time-domain analysis, the computed response of the structure from time history analysis with e.g. ADINA agree reasonably well with those of the frequency domain analysis using SASSI.

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