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ACCURACY OF THE SUBTRACTION MODEL USED IN SASSI

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ABSTRACT

In a recent letter to the U.S. Department of Energy (DNFSB, 2011), the Defense Nuclear Facilities Safety Board (DNFSB) raised some concerns regarding the technical accuracy and proper validation of the Subtraction Model implemented in different versions of the SASSI program. The DNFSB reported that the response transfer functions calculated using the Subtraction Model for frequencies above 10 Hz exhibited peaks and valleys, while those generated by the Direct Model were smooth and more reasonable. The DNFSB stated that the in-structure response spectra (ISRS) calculated using the Subtraction Model might become in some instances un-conservative for certain frequency ranges.

The purpose of this paper is to provide a brief background on several impedance modeling schemes used in SASSI, to examine the accuracy of three such schemes (the Direct, Subtraction, and Modified Subtraction Models) and to provide additional guidance in applying these models to large-scale SSI problems. The conclusions presented in this paper are based on the results of two test problems analyzed in this study.

BACKGROUND

Direct Model

The SSI analysis methodology employed in SASSI (Lysmer, et al, 1981), referred to as the Flexible Volume Method (FVM) (Tabatabaie, 1982 and Tajirian, 1981), is based on the observation that the solutions to scattering and impedance problems in the general sub-structuring approach can be greatly simplified if the interactions are considered over a volume rather than a boundary. In the Flexible Volume Method, the dynamic stiffness of the structure is reduced by the corresponding properties of the excavated soil volume, which is retained within the halfspace (i.e. horizontally layered). As a result, the scattering problem associated with a ground cavity reduces to that of the free-field ground response problem, while the impedance problem reduces to a point load solution in a horizontally layered system. The calculation of the impedance matrix thus involves performing an inversion of a full flexibility matrix associated with all flexible volume interaction nodes developed from a point load solution in a layered system. By imposing common degrees-of-freedom between the excavated soil model and the impedance model (i.e. free-field point load model), the compatibility of the displacements at all interaction nodes -- including those within the excavated soil volume -- are satisfied. This ensures accurate and stable results that converge to the true solution as the mesh refinement is increased. This is the Direct Modeling (DM) of impedance calculation (see Figure 1.a).

Skin Model

The Direct Model involves the inversion of a large, fully-populated, complex-valued flexibility matrix whose size grows approximately by the power of 3 as the dimensions and/or mesh refinement of the embedded foundation model increase in three-dimensional problems. To reduce the numerical effort involved in inverting a large flexibility matrix, the original SASSI program includes an alternative scheme for calculating the impedance matrix, referred to as the Skin Model (Tabatabaie, 1982). In this modeling scheme, only the degrees-of-freedom associated with the interaction nodes on the excavation skin (referred to as interface nodes) are considered in the inversion of the flexibility matrix, significantly reducing the numerical effort required to calculate the impedance matrix (see Figure 1.b for the definition of interface, intermediate, and internal nodes).

In applying the Skin Model, it is not theoretically necessary to impose the compatibility of displacements at the internal nodes within the excavated soil volume. In reality, the internal nodes are fictitious and only included for mathematical convenience. The stiffness terms associated with the internal nodes are expected to cancel each other out when the dynamic stiffness of the excavated soil model is subtracted from the impedance matrix. The Skin Model imposes the compatibility of displacements at the interface nodes, but at the internal nodes this compatibility is only inferred. Because of the numerical difference in deriving the impedance matrix from the free-field point load solution and direct stiffness formulation of the dynamic stiffness of the excavated soil model, the Skin Model only provides acceptable impedance solutions if the cut-off frequency is set very low (i.e. to $V_s/12h$ or even lower, where V_s is the shear wave velocity of the foundation media and h is the smallest element size in the excavated soil model). As a result of this limitation, the Skin Model was not recommended for practical application. And as it has already been studied in detail (Tabatabaie, 1982) and remained largely unused, it will not be re-examined in this paper.

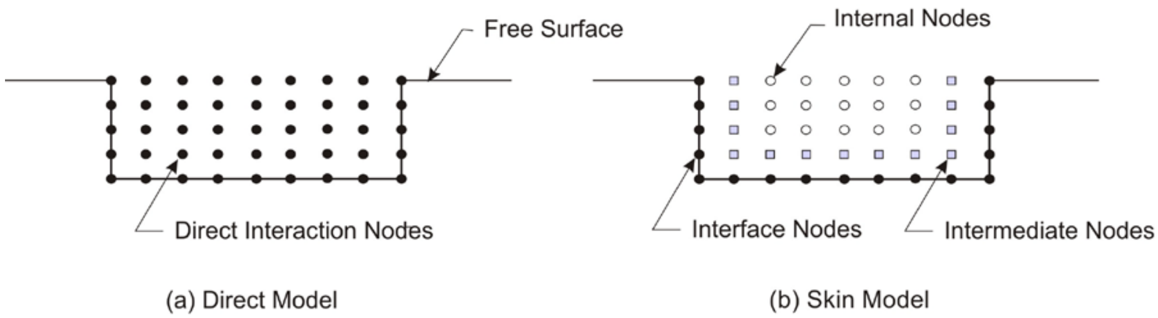


Figure 1. Illustration of Direct and Skin Models

Symmetric Impedance Model

To further reduce the size of the impedance matrix, Symmetric and Anti-Symmetric Impedance Models (SIM) that take advantage of the system's symmetry were also developed and incorporated into the original SASSI program (Lysmer, et al, 1981). These models, derived from the special application of point loads in a layered system, significantly facilitated the SSI analysis of structures with large embedded foundations. Because the derivation of the Symmetric and Anti-Symmetric Models is exact and fully validated, they will not be re-examined in this paper.

Rigid Impedance Model

Later attempts to further reduce the size of the impedance matrix led to the development of the Rigid Impedance Model (RIM) (MTR/SASSI, 2011). This model is based on the assumption that the response of a rigid foundation can be fully described by 6 degrees-of-freedom (3 translations and 3 rotations). Taking advantage of the foundation's rigidity, the size of the complex-valued flexibility matrix was reduced to a 6x6 matrix, thus completely eliminating the need to invert a large flexibility matrix. Because this feature is not available in the original SASSI program, as well as limited to foundations with rigid base slabs, it will not be discussed in this paper.

Subtraction Model

The Subtraction Model (SM) is an alternative modeling scheme that was later adopted by SASSI for solving impedance problems. In this scheme, only the interface nodes are considered as interaction nodes (i.e. the compatibility of displacements is no longer imposed at all interaction nodes within the soil volume) (see Figure 2.a). In some respects, this model is similar to the Skin Model with one exception: the compatibility of displacements at the internal nodes is considered in the Skin Model, whereas in the Subtraction Model it is not imposed. The Subtraction Model gained popularity because, like the Skin Model, it significantly reduced the numerical effort involved in calculating the impedance matrix for large embedded structures. However, it suffers from the same issues of numerical inaccuracy that were originally observed in the Skin Model (Tabatabaie, 1982). These issues, as raised by the DNFSB, are further explored in this paper.

Modified Subtraction Model

The Modified Subtraction Model (MSM) is a proposed improvement over the Subtraction Model. According to this modeling scheme, the compatibility of displacements, in addition to the skin nodes, is imposed at the internal nodes located on the free-field surface by specifying those nodes as interaction nodes (see Figure 2.b). The accuracy of the Modified Subtraction Model is further studied in this paper.

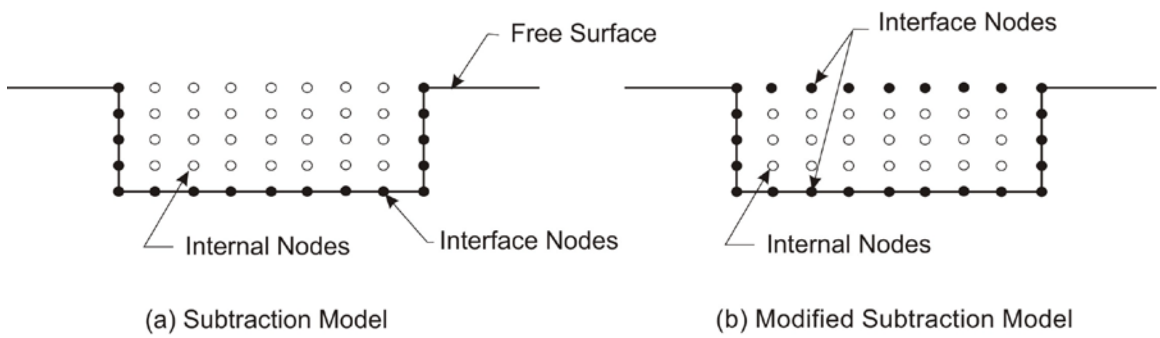


Figure 2. Illustration of Subtraction and Modified Subtraction Models

SASSI PROGRAM

The accuracy of different impedance modeling schemes in SASSI was investigated using MTR/**SASSI** (2011). This version of SASSI makes no distinction between the Direct, Skin, Subtraction, and Modified Subtraction Models, or any combination of the interaction node sets used to develop the impedance matrix. Because they are considered modeling schemes, the user simply specifies sets of interaction nodes (interface nodes) for which the compatibility of displacements between the excavated soil nodes and free-field point load model is imposed. These sets of interaction nodes are selected from among the excavated soil nodes, the balance of which is obtained automatically by the program and designated as “internal nodes.” The compatibility of displacements is not imposed at the internal nodes.

The Direct Model, being the most accurate, specifies all the nodes within the excavated soil model as interaction nodes. The Subtraction Model, being the least accurate, specifies only the nodes on the excavation skin as interaction nodes. Other modeling schemes, such as the Skin and Modified Subtraction Models, specify more of the nodes within the excavated soil model as interaction nodes. In MTR/**SASSI** any impedance modeling scheme that does not impose the compatibility of displacements at all internal nodes (such as the Skin, Subtraction, and Modified Subtraction Models) is actually a subset of the Direct Model with incompatible displacements.

To investigate the potential technical issues raised by the DNFSB regarding the Subtraction Model, two test problems are examined in this paper. The first is a benchmark problem that compares the results of the Direct, Subtraction, and Modified Subtraction Models in terms of scattering and impedance solutions against those of published literature. The second represents a simplified model of a nuclear power plant (NPP) sized structure analyzed for a standard soil site in the Western United States (WUS) and a hard rock site in the Central and Eastern United States (CEUS). The results of the second model in terms of computed transfer functions, maximum accelerations, and response spectra obtained from different modeling schemes are compared at several key locations in the structure. Details of the two test problems and a detailed discussion of the results are provided in Tabatabaie, 2011. The results are briefly discussed in this paper.

TEST 1: SCATTERING AND COMPLIANCE FUNCTIONS FOR A RIGID FOUNDATION

The scattering and compliance functions for a rigid, massless, cylindrical foundation fully embedded in a uniform halfspace are used to examine the accuracy of the Direct, Subtraction, and Modified Subtraction Models for seismic SSI analysis. Available solutions to this problem are reported in Day (1977) and Apsel and Luco (1987). This is considered a "benchmark problem."

Problem Description

Figure 3 shows the foundation model and properties. The foundation has an embedment ratio of R/H = 1, where R and H are the radius and depth of the foundation, respectively. The properties for this problem are dimensionless. The halfspace has a damping of 1% for shear waves and 0.5% for compression waves.

The computer program MTR/**SASSI** is used to calculate the foundation scattering and compliance functions via the Direct, Subtraction, and Modified Subtraction Models. The scattering properties of the foundation are obtained for vertically propagating plane shear waves with control motion assigned at the free-field surface, and the results are compared to those reported in Day (1977). The foundation compliance functions include the vertical, horizontal, rocking, and coupled horizontal-rocking components. The calculated foundation compliance functions are compared with those reported in Apsel and Luco (1987). The scattering and compliance functions refer to the bottom center of the foundation.

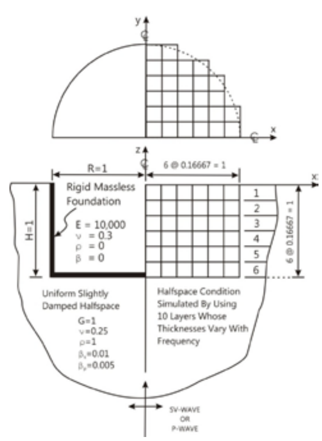


Figure 3. Foundation Model and Properties

The problem is analyzed for three interaction node sets corresponding to the Direct, Subtraction, and Modified Subtraction Models. The passing frequency of the model (f_{pass} = V_s/5h, where h is the largest soil element size and V_s is the minimum shear wave velocity of the soil medium) is 1/5/0.16667 = 1.2 Hz. Analysis is performed to a frequency cut-off of f_{max} = 1.27 Hz, which is slightly higher than the passing frequency of the model. Because the problem is dimensionless, the results are expressed in terms of dimensionless frequency parameter, a_o, which is described as the ratio of the foundation dimension to the wave length of wave propagation: $a_o = 2\ \pi\ R\ /\ \lambda$, where R is the foundation radius, λ is wave length, and ω is circular frequency. By plugging $\lambda = V_s\ /\ f$ and $\omega = 2\pi f$ into the above equation, the maximum value of a_o corresponding to the cut-off frequency is about 8.

Discussion of Results

The results of the scattering solutions (horizontal and rocking transfer functions) for the Direct, Subtraction, and Modified Subtraction Models are shown in Figure 4, where they are compared against those of published solutions. The results of the compliance functions are compared against those of the published results in Figure 5, Figure 6, Figure 7, and Figure 8 for the vertical, horizontal, rocking, and coupled horizontal-rocking components, respectively. Note that the compliance functions are normalized and rendered dimensionless by multiplying them by G·R = 1.

A comparison of the scattering solutions for vertically propagating SV-waves using the Direct and Modified Subtraction Models shows excellent agreement with those reported in the literature for all frequencies up to the maximum a_o value of 8 (see Figure 4). The results for the Subtraction Model, however, start to deviate from the published results at about a_o = 3.

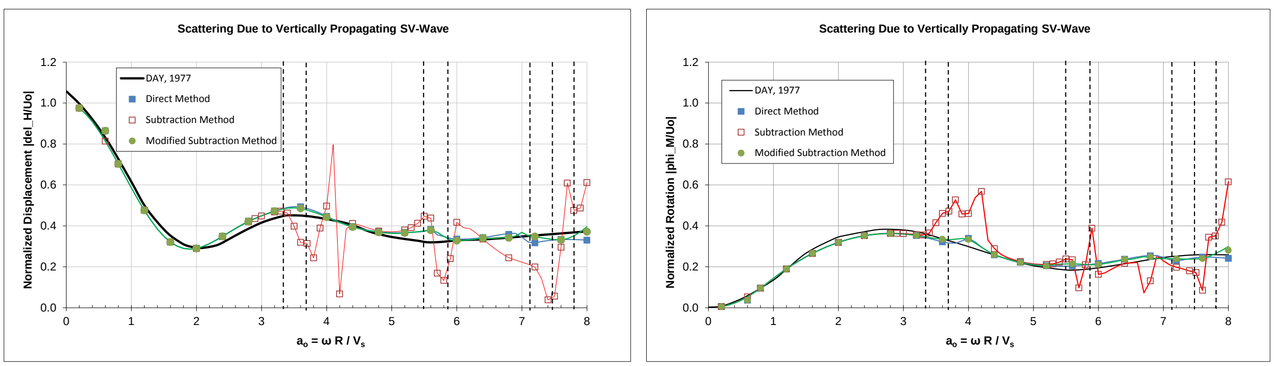


Figure 4. Foundation Scattering Response due to Vertically Propagating SV-Waves

A comparison of compliance functions obtained from MTR/**SASSI** using the Direct and Modified Subtraction Models also shows excellent agreement with the published results for all four components (i.e. vertical, horizontal, rocking, and coupled horizontal-rocking -- see Figure 5 through 8, respectively). Again, the results for the Subtraction Model start to deviate from the published results at a_o = 3.

As seen in Figure 5 through Figure 8, the compliance functions calculated using the Subtraction Model show a number of peaks and valleys at $a_o > 3$, causing significant departures from the target solution. These peaks and valleys are generally indicative of the wave energy trapped in the SSI model. Because the Subtraction Model does not constrain the internal nodes of the excavated soil model to the free-field point load model to satisfy displacement compatibility, it is reasonable to suspect that the energy entrapment occurs within the excavated soil model.

To better understand the deviation of the Subtraction Model's results from those of the target solution, the horizontal and vertical modes of the excavated soil model, restrained on the bottom and all four sides, are calculated using MTR/**SASSI**. These modes, which correspond to the peaks of the horizontal and vertical transfer functions calculated at the top of the soil model from input in the horizontal and vertical directions, respectively, are shown in Figure 9.a. An examination of these modes shows some correlation to the observed peaks and valleys in the scattering and compliance functions calculated using the Subtraction Model (see Figure 4 through Figure 8). By imposing the compatibility of displacements at the free-field surface nodes (i.e. the Modified Subtraction Model), these anomalies disappear, and the calculated results show good agreement with the target solution at all frequencies. One may suspect that this improvement is the result of shifting the modes of the soil model to frequencies beyond the frequency of interest by further restraining the top nodes. An examination of the modes of the soil model restrained on all sides, including the bottom and top (see Figure 9.b), reveals a shift to somewhat higher frequencies; nevertheless, they still remain within the frequency range of interest. And because these modes have no effect on the results of the Modified Subtraction Model, it is difficult to imagine that they are solely responsible for the spurious modes observed in the Subtraction Model results. Perhaps in this particular case, the generation of artificial surface waves at the free-field boundary at the top gives rise to additional wave reflections, which may result in energy being trapped within the soil model when the surface nodes are not constrained to the free-field point load model.

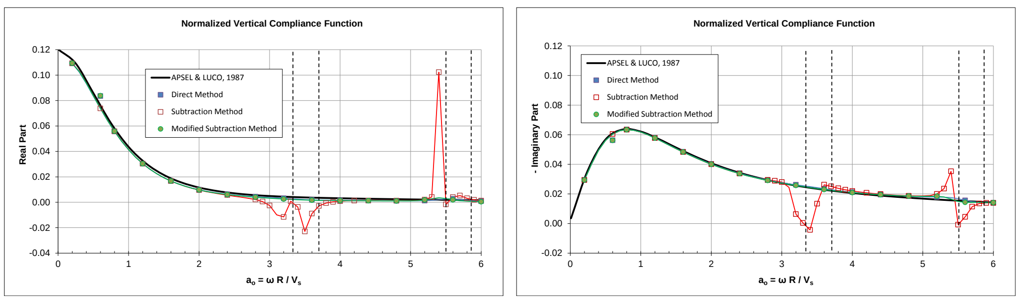


Figure 5. Normalized Foundation Compliance Functions, Vertical Component

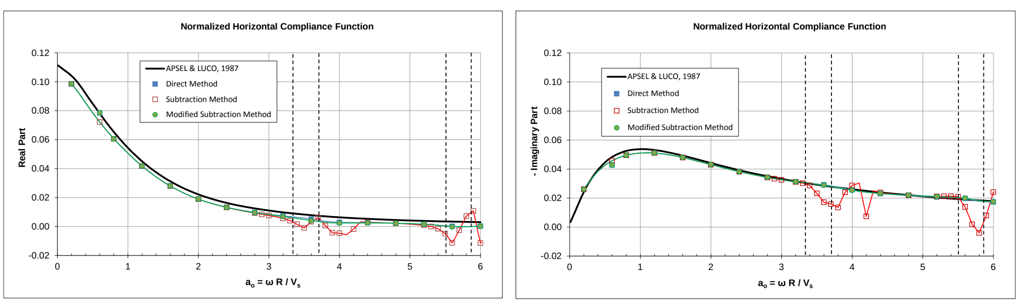


Figure 6. Normalized Foundation Compliance Functions, Horizontal Component

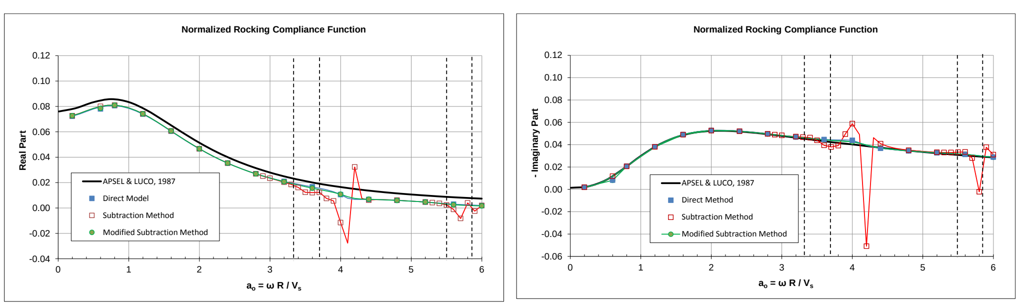


Figure 7. Normalized Foundation Compliance Functions, Rocking Component

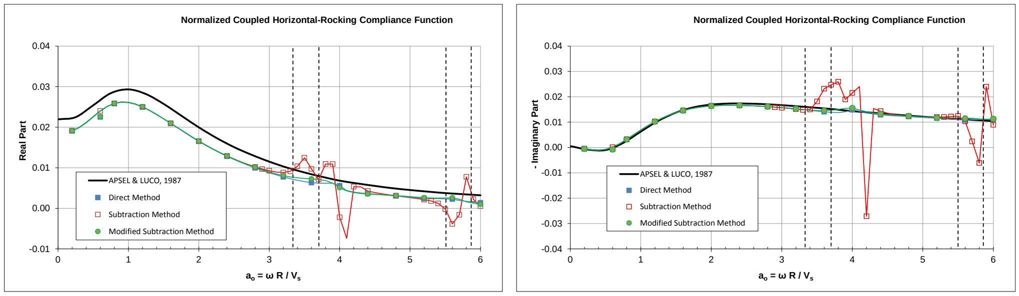
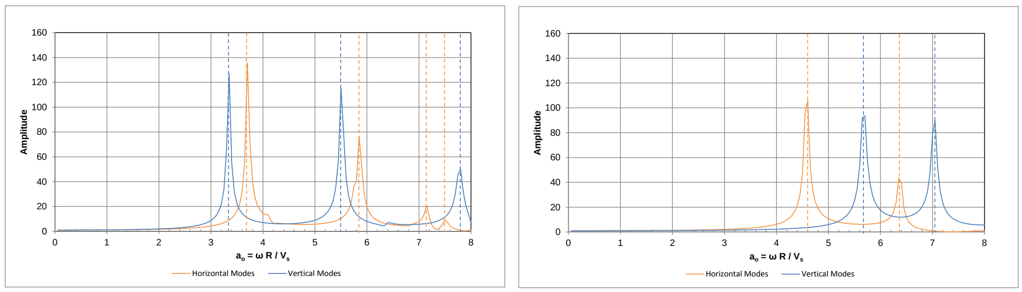


Figure 8. Normalized Foundation Compliance Functions, Coupled Horizontal-Rocking Component



(a) Model Constraints: Four Sides & Bottom (b) Model Constraints: Four Sides, Bottom & Top

Figure 9. Horizontal and Vertical Modes of Excavated Soil Model

TEST 2: SEISMIC RESPONSE OF A SHEAR-BOX STRUCTURE

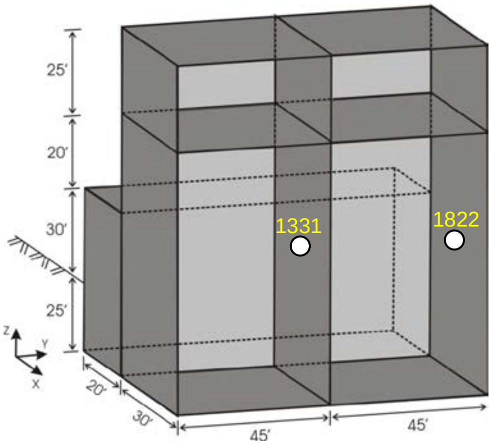
The purpose of this problem is to verify the accuracy of the Direct, Subtraction, and Modified Subtraction Models using MTR/**SASSI** to calculate the seismic SSI response of NPP structures. A simplified model of a typical NPP-sized structure is used to obtain and compare the results.

Problem Description

The problem consists of a concrete shear-box structure. A one-half cutaway view of the structure is shown in Figure 10. The structure is 100 × 180 feet in plan dimensions, with a vertical height of 100 feet and an embedment of 25 feet below the ground surface. The structure is modeled by 4-node plate/shell elements representing the basemat, walls, partitions, floors, and roofs. The excavated soil model consists of 8-node solid elements. The structure properties are shown in Figure 10.

Acceleration time histories and 5%-damped acceleration response spectra of input motion in the global x-, y- and z-directions are shown in Figure 11 for the standard soil site and in Figure 12 for the hard rock site. The standard soil site spectra are similar to the US NRC Reg. Guide 1.6 spectra. The hard rock spectra are typical high-frequency hard rock motions for the Central and Eastern United States.

The structure is subjected to vertically propagating P-, SV- and SH-waves with control motion specified at the free-field ground surface. The control motion consists of three orthogonal components, specified in terms of acceleration time histories in the global x-, y- and z-directions. The x- and z-components are associated with the SV- and P-waves, and the y-component is associated with the SH-wave propagation. Two analysis cases are considered: one corresponding to a generic soil site with standard input motion (WUS type motion) and another corresponding to a generic rock site with high-frequency motion (CEUS type motion).



Soil Case 1 (WUS Site):
Uniform semi-infinite halfspace
γ = 0.120 kip/ft^3
Vs = 825 ft/sec
Vp = 2,021 ft/sec
βs = 0.03
βp = 0.01

Structure:
Concrete members
γ = 0.210 kip/ft^3
E = 800,000 kips/ft^2
ν = 0.17
β = 0.04

Soil Case 2 (CEUS Site):
Top soil layer (25-ft thick)
γ = 0.120 kip/ft^3
Vs = 1,250 ft/sec
Vp = 3,061 ft/sec
βs = 0.03
βp = 0.01

Underlying halfspace
γ = 0.150 kip/ft^3
V_s = 10,000 ft/sec
V_p = 20,000 ft/sec
β_s = 0.005
β_p = 0.005

Figure 10. Structure & Soil Model Configuration and Properties

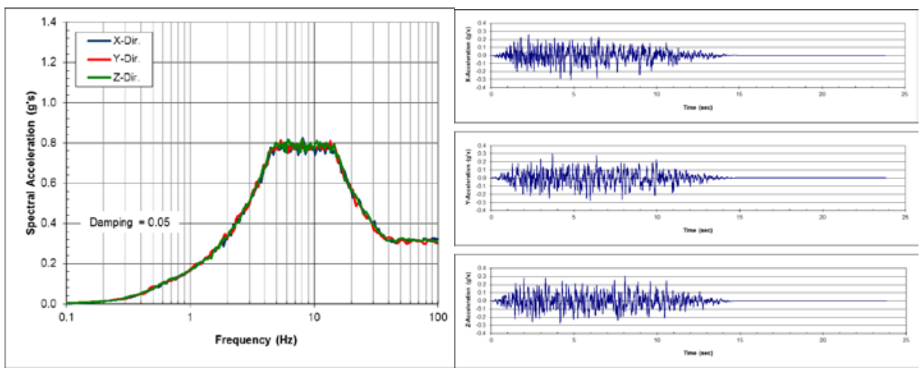


Figure 11. Response Spectra and Time Histories of Reference Input Motions, WUS Soil Site

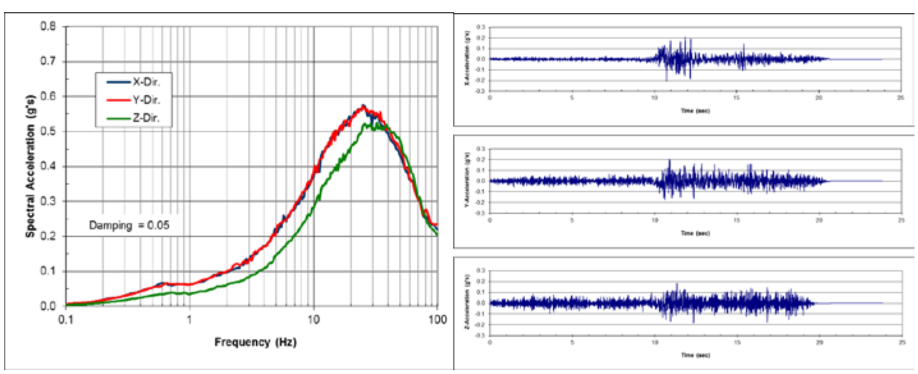
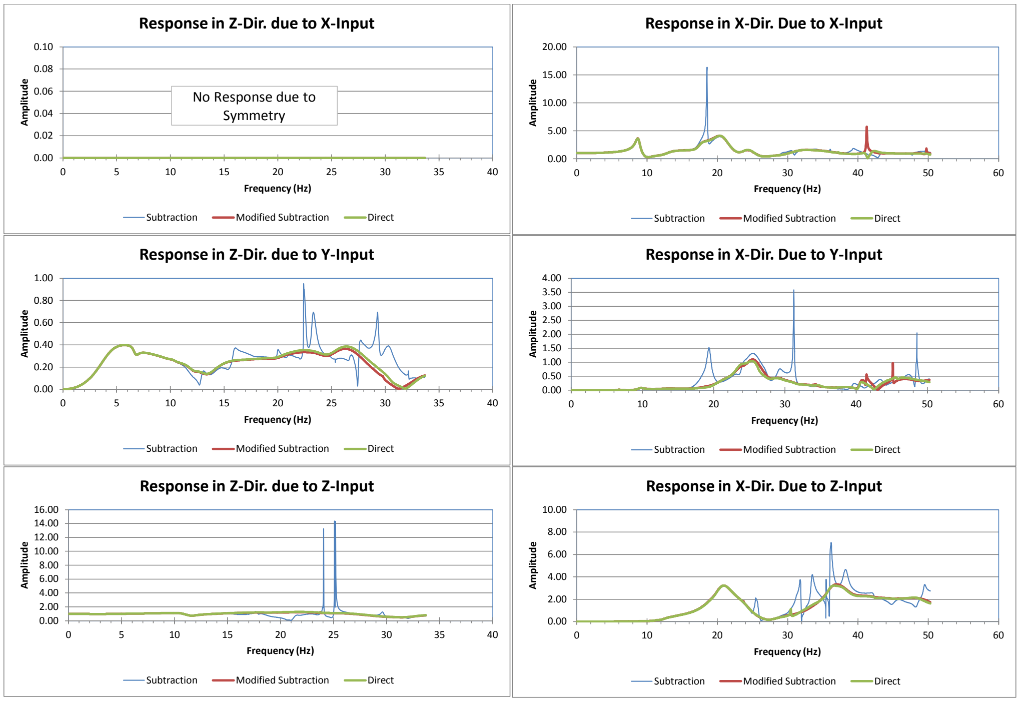


Figure 12. Response Spectra and Time Histories of Reference Input Motions, CEUS Hard Rock Site

Discussion of Results

Typical transfer functions calculated from the Direct, Subtraction, and Modified Subtraction Models are compared in Figure 12.a for the standard soil site and in Figure 12.b for the hard rock site. As these figures show, the transfer functions for the Direct and Modified Subtraction Models are smooth for the entire frequency range, showing excellent agreement for frequencies below 50 Hz. But the transfer functions for the Subtraction Model begin to deviate from the Direct Model's solutions at frequencies above 15 Hz, exhibiting numerous peaks and valleys, particularly at higher frequencies.

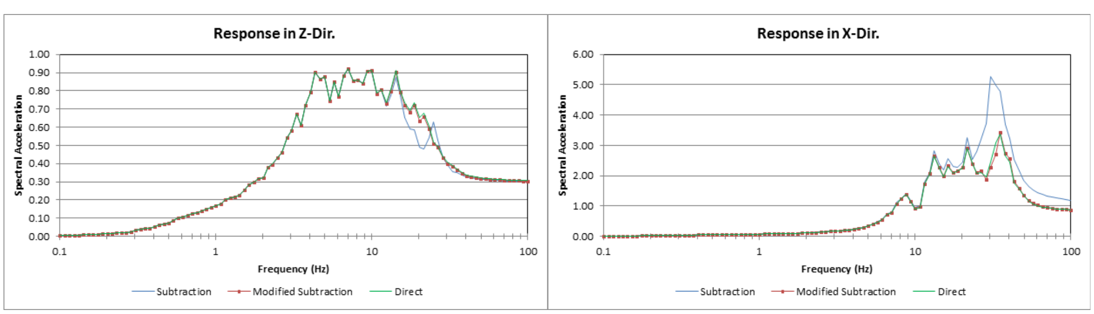


(a) Standard Soil Site (Node 1822) (b) Hard Rock Site (Node 1331)

Fig 12 - Comparison of Transfer Functions

Typical 5%-damped acceleration response spectra calculated from the Direct, Subtraction, and Modified Subtraction Models at Node 1822 for the standard soil site and at Node 1331 for the hard rock site are compared in Figure 13.a and Figure 13.b, respectively. For the location of output nodes, refer to Figure 10. The spectra are calculated from input motions applied in three directions. The results show good agreement between the Direct and Modified Subtraction Models. But again, the results of the Subtraction Model deviate from those of the Direct Model at frequencies above 10 Hz. This is consistent with the results of the transfer functions discussed above. The results indicate that the calculated spectra from the Subtraction Model can be lower than those of the other two models at certain frequency ranges (e.g. see Figure 13.a).

The calculated maximum accelerations at several select nodes in the structure obtained with the Subtraction Model vary from those of the Direct Model by about 88-127% for the standard soil site and 87-135% for the hard rock site. The results of the Modified Subtraction Model in terms of the maximum acceleration responses were found to be within 1% of the Direct Model's results.



(a) Standard Soil Site (Node 1822) (b) Hard Rock Site (Node 1331)

Figure 13. Comparison of 5%-Damped Acceleration Response Spectra

CONCLUSIONS

Based on an examination of the results in this study, the scattering and compliance functions derived for a rigid, massless, embedded cylindrical foundation using the Subtraction Model are only found to be accurate up to $a_o = 3$, where $a_o = \omega R / V_s$ and R is the equivalent foundation radius, V_s is shear wave velocity of soil media, and ω is circular frequency. When the value of a_o exceeds 3, the computed response transfer functions exhibit erroneous peaks and valleys that are believed to be associated with the wave energy trapped within the excavated soil model. For a typical NPP-sized model analyzed using the Subtraction Model, the departure of the transfer functions from the target solution occurs around 15 Hz for both the standard soil and hard rock sites. The impact of the transfer function departure on the final results (such as maximum acceleration values and in-structure response spectra) is found to be significant at some locations in the structure.

When the compatibility of displacements is also imposed at the internal nodes located at the free-field surface (as in the Modified Subtraction Model), the transfer functions become smoother, and the erroneous peaks and valleys disappear for values of a_o up to about 8 (as they are examined in this paper). The results of the Modified Subtraction Model are found to be closer to those of the Direct Model for the test problems analyzed in this study.

In general, the use of the Subtraction Model should be limited to cases where $a_o < 3$. For cases where $a_o > 3$, the Subtraction Model should be used with caution as it may result in erroneous peaks and valleys in the calculated response transfer functions. The impact of these spurious modes on the final results can be significant, particularly if they are affected by the energy of input motion. The results of the Modified Subtraction Model are close to those of the Direct Model, validated using a benchmark problem for a_o values up to about 8 and compared for a typical NPP-sized model for both the standard soil and hard rock sites.

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